On the Distributional Effects of Monetary Shocks and Market Incompleteness*

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Abstract

I study the transmission of distortionary monetary policy shocks under incomplete markets. Using a heterogeneous agents general equilibrium model, I demonstrate that there is a unique fundamental stationary equilibrium, where the distribution of monetary holdings mirrors productivity, but infinite non-fundamental stationary equilibria for a given monetary base in the presence of a frictionless bonds market. Only financially constrained economies return to the fundamental stationary equilibrium after an unforeseeable monetary shock that redistributes monetary holdings, with aggregate effects on output and endogenous price stickiness along the transition. Financially developed economies display smaller distortions and negligible effects on aggregate variables, but monetary shocks create hysteresis by making the consequences of idiosyncratic shocks permanent. While partial market completion enhances welfare by enabling nearly perfect risk sharing, this improvement is limited by the irreversibility of the idiosyncratic shocks. Ultimately, distributional effects are irrelevant for monetary policy transmission to aggregate variables in developed economies but critical in poorer countries.

Keywords: Monetary Shocks, Distributional Effects, Market Incompleteness, Credit, Financial Development

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1 Introduction

There is growing evidence that monetary policy shocks produce distributional effects, sparking a debate over their relevance to central banks' choices. These shocks can affect relative income, benefit debtors at creditors' expense, and change asset prices, redistributing wealth between asset holders and non-holders. This is important given that, even in the U.S., only 21% of households held stocks directly, and only 1.1% held bonds as of 2022 according to the Survey of Consumer Finances (SCF). Moreover, households that own stocks and bonds,[1] directly or indirectly, are approximately twice as rich as the average. Although credit markets could mitigate these wealth shocks through risk-sharing, access remains limited, as roughly 20% of U.S. families lacked a credit card and 10% had at least one loan application rejected in the year before the SCF survey.

These distributional effects shape the transmission of monetary shocks. As Friedman (1969) noted, the random re-shuffling of money prevents an immediate return to the initial equilibrium allocations, even in a frictionless economy. Agents made richer by the shock would smooth consumption, and incomes would adjust to restore the long-run equilibrium. As a result, the resulting transition path should be sluggish, directly affecting policy effectiveness within the relevant monetary policy horizon. Along the transition, temporary shifts in consumption and income should arise, especially in the absence of financial tools enabling risk sharing. I investigate these claims by studying the transmission of distortionary monetary policy shocks and how financial development — in the form of access to credit — affects post-shock dynamics.

I develop a tractable general equilibrium model featuring a cash-in-advance friction and a monopolistically competitive goods market. There is perfect foresight and costless price adjustment. To analyze how the redistribution of monetary holdings affects inequality, I introduce productivity heterogeneity, which generates dispersion in monetary holdings, consumption, output, and prices. Then, a monetary shock generates a wedge in money holdings between equally productive agents. This model allows me to show that the introduction of one-period bonds suffices to significantly alter the post-shock dynamics by enabling risk-sharing, which largely offsets distributional effects in consumption and renders money nearly neutral in the aggregate.

First, I establish the existence of a unique fundamental stationary equilibrium, characterized by a wealth distribution that mirrors the productivity distribution, given the market structure and consumer preferences. However, for a given monetary base, any redistribution of monetary holdings across agents is compatible with a stationary

[1] Excluded retirement funds, savings, and foreign bonds.

equilibrium with borrowing from a frictionless bonds market. This is because agents with insufficient assets relative to their productivity become indebted and roll their debt indefinitely. Likewise, entrepreneurs whose assets are too high given their income decide to permanently maintain savings and receive interest payments. This way, one-period bonds work as perpetuities. Although these findings apply to any extrinsic wealth redistribution, monetary shocks provide a natural application.

Then, I investigate the dynamics after a one-time unforeseeable monetary shock that happens to an economy at the fundamental stationary equilibrium. I show that the distributional effects of monetary shocks indeed induce a more sluggish and distorted return to the fundamental stationary equilibrium in the absence of access to credit markets. On the other hand, and contrary to Friedman's hypothesis, if there is full enforcement of debt repayments, the economy does not return to the initial allocations, and the induced differences in monetary holdings become permanent. This implies that the effect of these shocks on wealth inequality can be persistent.

The model is closely related to the financial segmentation channel, proposed by Williamson (2008). In this paper, he assumes that households are either *connected* or *unconnected* to financial markets, with no possibility of moving between groups. By *connected*, he means that these households operate frequently in financial markets and, hence, are the first to be affected by monetary shocks. This heterogeneity is well-illustrated by the aforementioned low levels of financial asset ownership observed in the U.S. Throughout, I will use the same classification adopted by Williamson (2008).

For a sufficiently large positive (negative) monetary shock, connected households' consumption increases (decreases) relative to the unconnected, but the revenues of the latter are higher (lower). This formalizes the mechanism proposed by Friedman (1969), leading to endogenous finite-time convergence to the long-run equilibrium in economies without borrowing. Unconnected (connected) households gradually absorb money that was initially idle, linking portfolio-related distributional effects to the more indirect, general equilibrium, income ones. As a result, consumption and wealth inequality move in the opposite direction of income inequality after the shock.

I allow the whole productivity distribution between connected and unconnected agents to differ. Since connected agents are more productive on average in the data, this allows me to capture a novel dimension of the disparity between both groups. If the monetary shock benefits, on average, poorer agents, it reduces inequality. However, the shock also creates a wedge between connected and unconnected agents *with the same productivity*. This wedge shrinks in the presence of a bonds market, which unambiguously improves welfare by allowing for risk sharing. Moreover, I show that allocative efficiency fluctuates if the connectedness status is correlated with productivity, as the beneficiaries from the shock cut their labor due to the wealth effect.

The paper's primary contribution is to examine how wealth redistribution unrelated to fundamentals affects the transmission of monetary shocks. I study transitional dynamics between stationary equilibria through endogenous mechanisms, offering a novel analysis of convergence properties that extends beyond the previous approaches in the literature. Moreover, I show that introducing a simple one-period bond fundamentally alters the transmission and improves welfare, by mitigating distortions through risk-sharing and resolving the idle cash balances problem. However, it leads to hysteresis in post-shock monetary holdings. Additionally, the model generates real effects and endogenous price stickiness through two novel mechanisms: (i) unconnected agents set lower prices than connected agents to restore real balances, and (ii) connected agents lower production and, thus, marginal costs after a positive shock.

These findings highlight differences between economies with well- and poorly-developed credit markets. In less developed economies, monetary policy induces stronger distributional effects on consumption and more output volatility. In contrast, widespread access to credit markets makes distributional effects irrelevant to monetary policy transmission to aggregate variables. Broader access to financial asset markets exposes more people to monetary policy risk but can make shocks less distortionary if sufficiently widespread. This underscores the need for robust credit markets to ease distortions from monetary shocks while balancing short-term stabilization with long-term policies to address inequality.

Lastly, I conduct a sensitivity analysis. First, I examine a negative monetary shock and CRRA utility, followed by the impact of different Frisch elasticities on shock transmission. I then vary access to financial asset markets, showing that higher access: (i) dilutes post-shock distortions under money-supply targeting, but (ii) induces an inverse U-shaped effect of the shock on distributional distortions under interest rate targeting. I then derive implications for futures markets. Next, I show that transition dynamics depend on the relative size of individual vs. aggregate shocks and that the fraction of connected agents influences transition length in the bondless economy. Lastly, I analyze collateral constraints.

The paper is organized as follows. Section 2 develops the baseline model and analyzes its stationary equilibria. Section 3 examines post-shock dynamics without a bonds market, then introduces bonds, studies market equilibrium and the zero interest rate case, and conducts a welfare analysis. Section 4 presents the sensitivity analysis. Lastly, section 5 concludes. All proofs are presented in Appendix A, while Appendix B contains further graphs, and Appendix C, outstanding tables.

1.1 Related Literature

This paper contributes to two strands of the literature. First, it relates to the literature on the distributional effects of monetary policy, which shows that monetary shocks affect agents based on their income compositions, portfolios, financial market participation, and skill level (Hohberger et al., 2020; Coibion et al., 2017; Dolado et al., 2021). Distributional effects may also arise from the heterogeneity of price adjustment (Cravino et al., 2020; Baqaee et al., 2022), risk sharing mechanisms (Chiu and Molico, 2010; Rocheteau et al., 2018), and the regressive interaction between inflation tax and economies of scale in credit transactions (Erosa and Ventura, 2002). The empirical evidence on net effects remains mixed, with studies linking contractionary monetary shocks to rising inequality (Coibion et al., 2017; Furceri et al., 2018), while others find the opposite (Davtyan, 2016; Montecino and Epstein, 2015).

A key distinction in the literature is between direct and indirect, general equilibrium, transmission channels (Ampudia et al., 2018). Portfolio revaluations are a significant direct effect of monetary policy in several empirical studies (Doepke and Schneider, 2006; Saiki and Frost, 2014; Ampudia et al., 2018; Auclert, 2019). There is also substantial evidence in support of indirect income-related effects on wages, capital returns, skill premium, or employment fluctuations (Gornemann et al., 2016; Coibion et al., 2017; Dolado et al., 2021; Casiraghi et al., 2018). One of my contributions lies in integrating these direct and indirect channels by modeling the latter as an endogenous response to the former, as suggested in Friedman (1969).

Two papers closely related to mine are Williamson (2008) and Grossman and Weiss (1983). My model differs from Williamson's by assuming away goods market segmentation[2] and perfect competition while allowing agents to save money. This endogenizes the convergence process and enables the study of its properties and mechanisms. I also generalize his model by introducing borrowing between connected and unconnected agents,[3] keeping the absence of risk-sharing as a special case. Grossman and Weiss (1983) model open market operations through staggered bank withdrawals, generating a gradual transition. My framework, in contrast, allows for endogenous convergence, output responses, and risk-sharing, offering a broader perspective on monetary policy's distributional effects.

[2]In his paper, connected agents trade primarily among themselves in a competitive goods market, while unconnected agents operate in a partially separate market. Convergence to the stationary equilibrium happens asymptotically because money flows between submarkets, as matches across submarkets occur with some exogenous probability.

[3]In Williamson's framework, agents do not keep any savings and, although a credit market exists, only connected agents can access it.

Secondly, I contribute to the literature on incomplete markets, particularly on the role of debt. In the New-Monetarist tradition (Kiyotaki and Wright, 1993; Lagos and Wright, 2005), imperfect credit markets make money essential for transactions. If some agents hold idle balances while others are liquidity-constrained, borrowing could reallocate liquidity from low- to high-marginal utility agents (Berentsen et al., 2007). I examine how this reallocation mechanism influences the transmission of monetary shocks. Prior work in this tradition shows that idiosyncratic trading histories produce a non-degenerate money distribution, allowing monetary policy to act as risk-sharing by redistributing liquidity (Chiu and Molico, 2010; Rocheteau et al., 2018; Chiu and Molico, 2021). In contrast, my framework, which does not model monetary shocks as lump-sum transfers, produces the opposite result.

This paper also relates to Eggertsson and Krugman (2012), where a tightening of borrowing requirements generates distributional effects and depresses aggregate demand. In both models, the liquidity provided to constrained households eases distortions. Unlike theirs, my framework considers external monetary shocks, focusing on credit markets' role in risk-sharing and its implications for monetary policy transmission. Furthermore, access to liquidity through a frictionless credit market significantly completes the market as in Telmer (1993), practically undoing the heterogeneity in consumption. I extend this literature by characterizing post-shock dynamics and showing that financial development while improving welfare, can also introduce hysteresis in monetary holdings. This effect is, to the best of my knowledge, novel to the literature.

2 The Model

Consider an economy with a continuum of entrepreneurs with unit mass, who differ in their time-invariant productivity, z. The productivity follows a cumulative distribution function $\mathbb{F}(\cdot)$. Every entrepreneur produces an intermediate good through her own work and derives utility from the consumption of the final good. There is also a final good firm, which operates in a competitive market and produces a composite final good out of the intermediate goods produced by the entrepreneurs. Moreover, I assume that there is a market for riskless one-period pure-discount bonds.

I assume that entrepreneurs have identical preferences. Moreover, as in Williamson (2008), I define *connected agents* as those who frequently trade in financial markets, being, therefore, directly affected by monetary shocks. They correspond to a fraction $\eta \in (0,1)$ of the population, and their productivity is distributed according to the c.d.f. $F_c(\cdot)$. *Unconnected agents* are, naturally, affected indirectly by monetary shocks and correspond to a fraction $1-\eta$ of the population. Their productivity is distributed according to the c.d.f. $F_u(\cdot)$. Naturally, $\mathbb{F}(z) = \eta F_c(z) + (1-\eta)F_u(z)$. Finally, I denote $\eta_i = \eta$ for $i = c$ and $\eta_i = 1-\eta$ for $i = u$. For simplicity, I assume a common support for these distributions, and that connectedness status is fixed for each agent.

The timing of the model goes as follows:

1. All bonds, $b_{it}(z)$, purchased in the previous period reach maturity;

2. The entrepreneur with productivity z and connectedness status $i \in \{c, u\}$ starts with $m_{it}^{-}(z)$ units of money, and connected agents may receive an unanticipated and unforeseeable transfer (tax) $\tau m_{ct}^{-}(z)$ from the government, financed through money creation (destruction). Thus, $m_{it}(z) = (1 + \mathbb{1}_{i=c}\tau)(m_{it}^{-}(z) + b_{it}(z))$, where $\mathbb{1}_{i=c} = 1$ when the agent is connected, and $\mathbb{1}_{i=c} = 0$ otherwise;

3. The entrepreneur sets a price $p_{it}(z)$ for the good they produce. Given these prices, the final goods firm buys on credit the output of each entrepreneur, $y_{it}(z)$, and produces thereby a composite good Y_t;

4. Each entrepreneur decides, given the final good price, P_t, and bond price, q_t, how much of $m_{it}(z)$ to spend on consumption, $C_{it}(z)$, how much to save as idle cash, $s_{it}(z) \geq 0$, and how much to spend on bonds. Alternatively, they can sell bonds.

5. The final goods firm pays the entrepreneurs for the purchases made.

6. Sales revenues and the money unspent in the period will sum up to $m_{i,t+1}^{-}(z)$.

As usual, if the bond is bought, $b_{it}(z) > 0$; if it is sold, $b_{it}(z) < 0$. Moreover, monetary shocks are assumed to become immediately known by everyone whenever they take place, that is, before agents make any pricing and production decision for that period. The timing also implies that *entrepreneurs cannot benefit from current sales* because purchases are only paid for at the end of the period. Thus, there is a cash-in-advance (CIA) friction. Moreover, there is imperfect competition in the intermediate goods market, which allows for pricing decisions. I also assume that the monetary shock each entrepreneur receives is proportional to their current monetary holdings.[4] Importantly, as long as $\eta < 1$, we do not have helicopter drops of money.

Since $q_t > 1$ is not possible, $s_{i,t}(z) > 0$ can only happen if: 1) $q_t = 1$ or 2) the bonds market is shut down. I will also assume, henceforth, that, if the nominal interest rate is equal to zero, all the savings will still take place through the bonds market. Hence,

[4] I make this assumption for two reasons. First, it is fairly tractable given the pre-existing heterogeneity in cash holdings. Second, it seems more plausible to assume that monetary shock affects agents proportionally. For example, if a fall in the interest rate increases the price of a connected agent's assets, this valuation shock should be proportional to their asset holdings.

savings will never take the form of idle money if agents can buy and sell bonds. This can be rationalized as being the only choice that is robust to small upward trembles in the interest rate, which would always produce $s_{it}(z) = 0$ for any arbitrary $z \in [\underline{z}, \overline{z}]$.

An entrepreneur with productivity z and connectedness status $i \in \{c, u\}$ solves:

$$\max_{\{C_{it}(z), m^-_{i,t+1}(z), h_{it}(z), p_{it}(z), b_{i,t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[u(C_{it}(z)) - \gamma \frac{h_{it}(z)^{1+\zeta}}{1+\zeta} \right] \tag{1}$$

$$\text{subject to} \quad P_t C_{it}(z) + q_t b_{i,t+1} \leq m_{it}(z) \tag{2}$$

$$m^-_{i,t+1}(z) + P_t C_{it}(z) + q_t b_{i,t+1}(z) \leq m_{it}(z) + p_{it}(z) y_{it}(z) \tag{3}$$

$$m_{i,t+1}(z) = m^-_{i,t+1}(z) + b_{i,t+1}(z) \tag{4}$$

$$y_{it}(z) = z h_{it}(z) \geq D(p_{it}(z), P_t, Y_t) \tag{5}$$

$$b_{i,t+1}(z) \geq -l_t(z, m_{it}(z)) \tag{6}$$

$$C_{it}(z) \geq 0 \tag{7}$$

where $h_{it}(z)$ is labor; $D(p_{it}(z), P_t, Y_t)$ is the demand faced given the chosen price, $p_{it}(z)$; and $l_t(z, m_{it}(z)) \geq 0$ is the borrowing limit. Moreover, let $R_{it}(z) := p_{it}(z) y_{it}(z)$ be the entrepreneur's revenue. As usual, I assume that the utility function satisfies $u \in \mathcal{C}^2$, $u'(\cdot) > 0$, and $u''(\cdot) < 0$. I assume isoelastic labor disutility for the sake of tractability and that $\zeta \geq 0$. Also for simplicity, I assume linear technology, i.e. $y_{it}(z) = z h_{it}(z)$. Moreover, notice that transfers enter implicitly in the CIA constraint (2), since, if an agent receives the transfers at the beginning of time t, $m_{ct}(z) = (1+\tau) m^-_{ct}(z)$. Lastly, given that monetary shocks are unforeseeable, agents assume that $m^-_{i,t+1}(z) = m_{i,t+1}(z)$. The final good's firm faces the static problem:

$$\max_{\{y^D_{it}(z)\}_{z \in [\underline{z}, \overline{z}], i \in \{c,u\}}} Y_t = \left(\sum_{i \in \{c,u\}} \eta_i \int_{\underline{z}}^{\overline{z}} y^D_{it}(z)^{\frac{\epsilon-1}{\epsilon}} dF_i(z) \right)^{\frac{\epsilon}{\epsilon-1}} \tag{8}$$

$$\text{subject to} \quad P_t Y_t = \sum_{i \in \{c,u\}} \eta_i \int_{\underline{z}}^{\overline{z}} p_{it}(z) y^D_{it}(z) dF_i(z) \tag{9}$$

$$y_t(z) \geq 0 \quad \forall z \in [\underline{z}, \overline{z}] \tag{10}$$

where $y^D_{it}(z)$ is the demand for the intermediate good of the entrepreneur with productivity z and connectedness status $i \in \{c, u\}$. Moreover, I assume that $\epsilon > 1$. Finally, I define $\mathbb{T}$ as the set of periods at which the economy is in a particular equilibrium path, and $t_0 := \min \mathbb{T}$. The equilibrium path of this economy is defined as follows:

Definition 1 (Equilibrium path). *An equilibrium path is a series of intermediate goods, final good and bond prices* $\{\{p_{it}(z)\}_{z \in [\underline{z}, \overline{z}]}, P_t, q_t\}_{i \in \{c,u\}, t \in \mathbb{T}}$, *individual consumption bundles*

$\{C_{it}(z)\}_{z\in[\underline{z},\overline{z}],i\in\{c,u\},t\in\mathbb{T}}$, intermediate and final good outputs $\{\{y_{it}(z)\}_{z\in[\underline{z},\overline{z}]},Y_t\}_{i\in\{c,u\},t\in\mathbb{T}}$, individual labor $\{h_{it}(z)\}_{z\in[\underline{z},\overline{z}],i\in\{c,u\},t\in\mathbb{T}}$, individual monetary holdings and monetary base $\{\{m_{it}(z)\}_{z\in[\underline{z},\overline{z}]},M_t\}_{i\in\{c,u\},t\in\mathbb{T}}$, and net bonds holdings $\{b_{i,t+1}(z)\}_{z\in[\underline{z},\overline{z}],i\in\{c,u\},t\in\mathbb{T}}$ such that, for every $t\in\mathbb{T}$:

1. *Given $\{q_t,P_t\}_{t\in\mathbb{T}}$ and the initial $m_{t_0}(z)$, $\{C_{it}(z),p_{it}(z),y_{it}(z),h_{it}(z),b_{i,t+1}(z)\}_{t\in\mathbb{T}}$ solve the problem (1) of the entrepreneur with $z\in[\underline{z},\overline{z}]$ and $i\in\{c,u\}$;*
2. *Given prices for the intermediate and final goods, $\{\{p_{it}(z)\}_{z\in[\underline{z},\overline{z}]},P_t\}_{i\in\{c,u\},t\in\mathbb{T}}$, intermediate goods demand and final good output $\{\{y^D_{it}(z)\}_{z\in[\underline{z},\overline{z}]},Y_t\}_{i\in\{c,u\},t\in\mathbb{T}}$ solve the problem (8) of the final good firm;*
3. *The intermediate goods markets clear, i.e. $y^D_{it}(z)=y_{it}(z)$ for $z\in[\underline{z},\overline{z}]$, $i\in\{c,u\}$;*
4. *The final good's market clears, i.e. $\sum_{i\in\{c,u\}}\eta_i\int_{\underline{z}}^{\overline{z}}C_{it}(z)dF_i(z)=Y_t$;*
5. *The bonds market clears, i.e. $\sum_{i\in\{c,u\}}\eta_i\int_{\underline{z}}^{\overline{z}}b_{i,t+1}(z)dF_i(z)=0$;*
6. *The monetary base is owned by entrepreneurs, i.e. $\sum_{i\in\{c,u\}}\eta_i\int_{\underline{z}}^{\overline{z}}m_{it}(z)dF_i(z)=M_t$.*

2.1 Solution

The solution to the problem of the final good firm, (8), takes the standard form:

$$D(p_{it}(z),P_t,Y_t)=\left(\frac{p_{it}(z)}{P_t}\right)^{-\epsilon}Y_t\qquad\forall z\in[\underline{z},\overline{z}]\tag{11}$$

and the final good price is given by:

$$P_t=\left(\sum_{i\in\{c,u\}}\eta_i\int_{\underline{z}}^{\overline{z}}p_{it}(z)^{1-\epsilon}dF_i(z)\right)^{\frac{1}{1-\epsilon}},\tag{12}$$

which I will, henceforth, refer to as aggregate price. The entrepreneur with productivity $z\in[\underline{z},\overline{z}]$ and connectedness status $i\in\{c,u\}$ follows the consumption schedule:

$$C_{it}(z)\begin{cases}=(u')^{-1}\left(\beta\frac{P_t}{P_{t+1}}u'(C_{i,t+1}(z))\right) & \text{if } s_{i,t}(z)>0\\ =\frac{m_{it}(z)}{P_t} & \text{if } s_{i,t}=0\text{ and } b_{i,t+1}(z)=0\\ \le(u')^{-1}\left(\frac{\beta}{q_t}\frac{P_t}{P_{t+1}}u'(C_{i,t+1}(z))\right) & \text{otherwise}\end{cases}$$

where $(u')^{-1}(\cdot)$ is the inverse of the marginal utility function. This function is well defined because $u'(\cdot)$ is injective and continuous. The cases above correspond, respectively, to 1) partial depletion (that is, the monetary holdings are not fully spent) with idle cash, 2) full depletion (the consumer spends all her money currently), and 3) partial depletion with non-zero bond holdings. Notice that the first case can only happen if the bonds market is shut down, which corresponds to a fully imperfect financial system. Moreover, we obtain a strict inequality in the third case if, and only if, (6) binds. Furthermore, the price chosen by the entrepreneur is given by:

$$p_{it}(z) = \underbrace{\left(\frac{\epsilon}{\epsilon - 1}\right)}_{\text{(i)}} \underbrace{\frac{\gamma h_{it}(z)^{\zeta}}{z}}_{\text{(ii)}} \underbrace{\frac{P_{t+1}}{\beta u'(C_{i,t+1}(z))}}_{\text{(iii)}} \tag{13}$$

This equation implies that there is a markup, (i), over marginal costs, (ii), and a forward-looking component, (iii), to pricing decisions. To understand the intuition, notice that revenues affect how much money the entrepreneur carries to the next period. The value of this money is the value of relaxing the future budget constraint, which is directly related to the marginal utility and the aggregate price in the next period. Whenever consumption will be large in the future, the value of relaxing the next period's budget constraint is lower. Hence, the agent will choose a relatively high price to get lower current labor disutility. Furthermore, the value of relaxing the budget constraint in the future is decreasing on future aggregate prices.

Now, I define the revenue of an arbitrary entrepreneur relative to the average as:

$$\theta_{it(z)} := \frac{p_{it}(z) D(p_{it}(z), P_t, Y_t)}{P_t Y_t} = \left(\frac{p_{it}(z)}{P_t}\right)^{1-\epsilon}, \tag{14}$$

which can be interpreted as the equivalent of a market share in the continuous case, since $R_{it}(z) = \theta_{it}(z) M_t$. It takes values $\theta_{it}(z) \in (0, 1)$ if the revenue of the entrepreneur with productivity $z \in [\underline{z}, \overline{z}]$ and connectedness status $i \in \{c, u\}$ is below the average revenue, $\theta_{it}(z) = 1$ if it is equal to average, and $\theta_{it}(z) > 1$ if it is larger. I study now the stationary equilibria of this economy.

2.2 The Stationary Equilibrium

I now define a stable price stationary equilibrium. The term "stable price" aims to restrict attention to stationary equilibria where the monetary base is constant. In these monetary equilibria, $m_{it}(z) = m_{i,t+1}(z)$ and $p_{it}(z) = p_{i,t+1}(z)$ for every $z \in [\underline{z}, \overline{z}]$ and $i \in \{c, u\}$, $M_t = M_{t+1}$ and $P_t = P_{t+1}$. Let $\mathbb{T}^S$ be the set of periods in which the economy is at this kind of equilibrium. I define it as follows:

Definition 2 (Stable price stationary equilibrium)**.** *A stable price stationary equilibrium for this economy is a series of prices* $\{\{p_{it}(z)\}_{z\in[\underline{z},\overline{z}]}, P_t, q_t\}_{i\in\{c,u\},t\in\mathbb{T}^S}$*, consumption, labor and output allocations* $\{\{C_{it}(z), h_{it}(z), y_{it}(z)\}_{z\in[\underline{z},\overline{z}]}\}, Y_t\}_{i\in\{c,u\},t\in\mathbb{T}^S}$*, and bond holdings*

$\{b_{i,t+1}(z)\}_{z\in[\underline{z},\bar{z}],i\in\{c,u\},t\in\mathbb{T}^S}$ *which, given* $\tau = 0$ *for and* $t \in \mathbb{T}^S$*, solve* (1) *and* (8) *and make* $m_{it}(z) = m_{i,t+1}(z)$ *for every* $z \in [\underline{z}, \bar{z}]$ *and* $i \in \{c, u\}$ *and, thus,* $M_t = M_{t+1}$.

Notice that the above definition implies that real allocations and relative prices are constant in all periods in stable price stationary equilibria. Moreover, since all the stationary equilibria studied throughout the paper are stable price, I will, henceforth, call them simply "stationary equilibria". As will be shown in Proposition 1, there are infinite such equilibria that are compatible with a given monetary base, M_t. I will, therefore, refine this concept further by defining *fundamental stationary equilibria* as:

Definition 3 (Fundamental stationary equilibrium)**.** *A fundamental stationary equilibrium for this economy is a stable price stationary equilibrium where* $m_{it}(z) = R_{it}(z)$ *for every* $z \in [\underline{z}, \bar{z}]$*,* $i \in \{c, u\}$ *and* $t \in \mathbb{T}^S$*.*

A fundamental stationary equilibrium is a stationary equilibrium where differences in monetary holdings across agents reflect differences in their fundamentals, which boil down to differences in productivity given our assumptions. Intuitively, agents are just as rich as their capacity to make money allows. As a result, for every $z \in [\underline{z}, \bar{z}]$ and $t \in \mathbb{T}^S$, $m_{ct}(z) = m_{ut}(z)$, that is, connected and unconnected agents with the same productivity have the same monetary holdings. The proposition below confirms that such a fundamental stationary equilibrium exists and is unique.

Proposition 1. *There is a unique fundamental stationary equilibrium, which requires that* $b_{i,t+1}(z) = 0$ *for all* $z \in [\underline{z}, \bar{z}]$*,* $i \in \{c, u\}$ *and* $t \in \mathbb{T}^S$*. Moreover, given the fundamental stationary equilibrium distribution of monetary holdings, it is the only possible equilibrium. Lastly, for any function* $m_{ct}(\cdot) > 0$ *and* $m_{ut}(\cdot) > 0$ *defined over the domain* $[\underline{z}, \bar{z}]$ *and satisfying* $\sum_{i\in\{c,u\}} \eta_i \int_{\underline{z}}^{\bar{z}} m_{it}(z) dF_i(z) = M_t$*, if* $l_t(z, m_{it}(z)) = R_{it}(z)$ *for all* $z \in [\underline{z}, \bar{z}]$ *and* $i \in \{c, u\}$*, there is a unique (non-fundamental) stationary equilibrium with borrowing compatible with it. This equilibrium requires* $q_t = \beta$ *for every* $t \in \mathbb{T}^S$*.*

Apart from the uniqueness of the fundamental stationary equilibrium, Proposition 1 implies that infinite non-fundamental stationary equilibria exist for any given monetary base, as long as the borrowing limit requires only that agents can repay their debt at the beginning of the next period[5]. Any distribution of money is made permanent through borrowing. However, the equilibrium is unique for any given distribution. Thus, without financial frictions, no mechanism ensures convergence to a fundamental stationary equilibrium.

[5]This is a sufficient condition. In Subsection 4.9, I will show that a tighter collateral constraint can often produce the same result.

Intuitively, indebted agents indefinitely roll over their debt and pay the interest with their sales revenue. Hence, these one-time bonds work like perpetuities. To put it simply, under a borrowing limit that ensures the capacity of debt repayment in the next period, consumption should either decrease, increase, or stay constant for *all* agents according to their Euler equation. The next section investigates the case of an MIT monetary shock to an economy starting at the fundamental stationary equilibrium.

3 Transition Dynamics

In what follows, I consider an economy that starts at the fundamental stationary equilibrium at $t = 0$ and receives an MIT monetary shock at $t = 1$ in the fashion described at the beginning of Section 2. Whenever I refer to the stationary equilibrium, I use the notation X_0 for $X \in \{Y, P, M\}$ and $x_0(z)$ for $x \in \{m, b, C, p, y, h, R, \theta\}$, which does not depend on connectedness status by the definition of fundamental stationary equilibrium. I start by studying a baseline economy with perfect financial frictions and only after allow the bonds market to become operational to study how the transmission of the shock is affected by financial development.

3.1 Baseline Economy

In the baseline economy, I assume that no borrowing can take place, *i.e.*, $l_t(z, m_{it}(z)) = 0$ for $z \in [\underline{z}, \bar{z}]$ and $i \in \{c, u\}$. This amounts to shutting off the bond market completely. I assume that the central bank introduces (withdraws) $\eta\tau M_{c0} > 0$ (< 0) units of money in the economy at $t = 1$, where $M_{c0} := \int_{\underline{z}}^{\bar{z}} m_0(z) dF_c(z)$ is the average monetary holdings of connected agents in the stationary equilibrium. Moreover, let $\tau^A := \eta\tau M_{co}/M_0$ be the proportional aggregate shock. I also define M_t^C as the money in circulation at time t, that is, the amount of money that is demanded in the economy for transaction motive, and $M_t = (1 + \tau^A)M_0$ is the monetary base. I now consider an economy that receives a monetary shock operated through helicopter drops.

3.1.1 Helicopter Drops Of Money

When helicopter drops take place, each agent gets a proportional $\tau_H = \tau^A$ over their monetary holdings. It is easy to see that the only possible equilibrium is one in which all agents fully deplete their money. Essentially, relative monetary holdings are not distorted by the shock since every agent gets the same proportional shock. Thus, the economy goes immediately to the new fundamental stationary equilibrium, in which the monetary base is $M_t = (1+\tau^A)M_0$ for $t \in \{1, 2, ...\}$. The corollary below formalizes that. The proof can be found in A.

Corollary 1.1. *After helicopter drops of money, the economy goes immediately to the new fundamental stationary equilibrium.*

Since all agents fully deplete their resources, prices are given by:

$$P_t^H = (1+\tau^A)P_0 \qquad \text{and} \qquad p_t^H(z) = (1+\tau^A)p_0(z) \tag{15}$$

for $z \in [\underline{z}, \overline{z}]$, $i \in \{c, u\}$ and $t = \{1, 2, ...\}$, where the H superscript refers to the "helicopter drops equilibrium", and consumption is identical to the consumption level in the initial stationary equilibrium level, that is, $C_{it}^H(z) = C_0(z)$ for $t = \{1, 2, ...\}$. Thus, a monetary shock implemented through helicopter drops is neutral, since prices immediately rise/fall uniformly and enough to put the economy at the new fundamental stationary equilibrium already at $t = 1$.

3.1.2 Uneven Access To The New Money

Now, I assume that the connected agents are the first to have their monetary holdings affected by the monetary shock. With some abuse of notation, I denote the agents who are made richer, in relative terms, by the monetary shock as *high-cash* and the ones that are made relatively poorer as *low-cash*. I denote the former with subscript h and the latter with subscript l. Naturally, if $\tau > 0$, then $h = c$, $m_{h1}(z) = m_{c1}(z) = (1+\tau)m_0(z)$ and $m_{l1}(z) = m_{u1}(z) = m_0(z)$ for any arbitrary $z \in [\underline{z}, \overline{z}]$; whereas $h = u$, $m_{l1}(z) = m_{c1}(z) = (1+\tau)m_0(z)$ and $m_{h1}(z) = m_{u1}(z) = m_0(z)$ when $\tau < 0$. Moreover, I define:

$$\overline{U}_t^{GAP} = \left(\frac{\sum_{i\in\{c,u\}} \eta_i \int_{\underline{z}}^{\overline{z}} z^{\epsilon-1} \frac{u'(C_{it}(z))^{\epsilon-1}}{h_{it}(z)^{\zeta(\epsilon-1)}} dF_i(z))}{\sum_{i\in\{c,u\}} \eta_i \int_{\underline{z}}^{\overline{z}} z^{\epsilon-1} \frac{u'(C_{i0}(z))^{\epsilon-1}}{h_{i0}(z)^{\zeta(\epsilon-1)}} dF_i(z)} \right)^{\frac{1}{\epsilon-1}}, \tag{16}$$

which captures the deviation, at a given moment, of a kind of weighted mean of the ratio of marginal utility of consumption over marginal labor disutility relative to the stationary equilibrium value, where the weights are a function of productivity. This expression is well-defined since the utility function is assumed to be equal for all agents, allowing for comparison across them. It will be useful to also define an analogous individual-level measure as:

$$U_{it}^{GAP}(z) = \left(\frac{z^{\epsilon-1} \frac{u'(C_{it}(z))^{\epsilon-1}}{h_{it}(z)^{\zeta(\epsilon-1)}}}{z^{\epsilon-1} \frac{u'(C_{i0}(z))^{\epsilon-1}}{h_{i0}(z)^{\zeta(\epsilon-1)}}} \right)^{\frac{1}{\epsilon-1}}. \tag{17}$$

In the following proposition, I characterize the dynamics after the shock.

Proposition 2. *After a monetary shock, there is a certain time $T < \infty$ in which the economy converges to the new fundamental stationary equilibrium. For $t = \{T, T+1, ...\}$, $p_{it}(z) = p^H(z, (1+\tau^A)M_0)$ for $z \in [\underline{z}, \overline{z}]$ and $i \in \{h, l\}$, $P_t = P^H((1+\tau^A)M_0)$, and $Y_t = Y_0$. Moreover, for $t = \{T+1, T+2, ...\}$, $C_{it}(z) = C_{i0}(z)$ and $m_{it}(z) = (1+\tau^A)m_0$ for all $z \in [\underline{z}, \overline{z}]$ and $i \in \{h, l\}$. Along the transition, that is, for $t = \{1, ..., T-1\}$, we have $C_{lt}(z) \leq C_{ht}(z)$, $\theta_{lt}(z) \geq \theta_{ht}(z)$, $p_{lt}(z) \leq p_{ht}(z)$, $y_{lt}(z) \geq y_{ht}(z)$, $R_{lt}(z) \geq R_{ht}(z)$, $m_{l,t+1}(z) \leq m_{h,t+1}(z)$, and $m_{h,t+1}(z) - m_{ht}(z) \leq m_{l,t+1}(z) - m_{lt}(z)$ for all $z \in [\underline{z}, \overline{z}]$, with strict inequality whenever the high-cash agent does not fully deplete. Besides, low-cash agents are always more likely to fully deplete than their high-cash counterparts. Finally:*

$$1 + \pi_{t+1} = \overline{U}_{t+1}^{GAP}, \tag{18}$$

and

$$\frac{\theta_{it}(z)}{\theta_{i0}(z)} = \left(\frac{U_{i,t+1}^{GAP}(z)}{\overline{U}_{t+1}^{GAP}} \right)^{\epsilon - 1}. \tag{19}$$

Proposition 2 implies that the economy eventually reaches the new fundamental stationary equilibrium, where the allocation is identical to the initial one, but prices are different. To understand this, notice that (19) means that the "market share" of a given entrepreneur, $\theta_{it}(z)$, will be higher (lower) whenever her ratio of marginal utility of consumption to labor disutility in the next period will be higher (lower) than the weighted mean, $\overline{U}_{t+1}^{GAP}$. This means that *artificially* poorer agents — *i.e.*, whose real balances fall below their fundamental level — will *tend* to have a higher market share than their fundamentals would suggest[6].

The intuition is that lower future consumption raises the value of holding money, leading poorer agents to set lower prices and work harder than their high-cash counterparts. This increases the marginal disutility of labor, partially offsetting the effect. Consequently, price and output differences emerge among agents with identical productivity due to a wealth effect, which gradually reduces disparities in money holdings along the transition path. This mechanism, akin to Friedman (1969), drives faster monetary accumulation for low-cash agents, reducing inequality.

Finally, (18) suggests that inflation (or deflation) persists while the weighted mean marginal utility deviates from its fundamental level. When distortions are large, inflation is higher. If $\overline{U}_{t+1}^{GAP} > 1$, the marginal future value of money is high on average. Then, prices would tend to be lower than their final stationary equilibrium level since

[6] What is meant with the word "tend" here is that some artificially poorer agents can still have a lower market share than in the stationary equilibrium if the rise in her future marginal utility is still lower than the rise in the weighted mean, $\overline{U}_{t+1}^{GAP}$, especially if marginal labor goes up enough. As will be shown later, log-utility (and homothetic utility functions in general) rules that possibility out.

many agents charge lower prices to replenish their real balances. As distortions shrink, future prices approach their stationary equilibrium level, resulting in inflation.

For the sake of tractability[7], I assume, henceforth, a logarithmic utility function, *i.e.* $u(\cdot) = \log(\cdot)$. Before I characterize the fundamental stationary equilibrium under the new specification, I define, respectively, the following aggregate and connectedness status-specific measures of productivity:

$$\mathcal{Z} := \left(\sum_{i \in \{c,u\}} \eta_i \int_{\underline{z}}^{\overline{z}} z^{\frac{\epsilon-1}{\epsilon}} dF_i(z) \right)^{\frac{\epsilon}{\epsilon-1}} \qquad \text{and} \qquad \mathcal{Z}_i := \left(\int_{\underline{z}_i}^{\overline{z}_i} z^{\frac{\epsilon-1}{\epsilon}} dF_i(z) \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{20}$$

where $i \in \{c, u\}$. The characterization of the log-specific fundamental stationary equilibrium follows as a corollary to Proposition 1.

Corollary 1.2. *For log-utility, for every $t \in \mathbb{T}^S$, aggregate output and final good prices are:*

$$Y_t = \mathcal{Z} \left(\frac{\epsilon - 1}{\epsilon} \right) \frac{\beta}{\gamma} \qquad P_t = \left(\frac{\epsilon}{\epsilon - 1} \right) \frac{\gamma}{\beta} \frac{1}{\mathcal{Z}} M_t \tag{21}$$

and relative revenues, monetary holdings, consumption, and prices are given by:

$$\theta_{it}(z) = \frac{z^{\frac{\epsilon-1}{\epsilon}}}{\mathcal{Z}^{\frac{\epsilon-1}{\epsilon}}} \qquad m_{it}(z) = \frac{z^{\frac{\epsilon-1}{\epsilon}}}{\mathcal{Z}^{\frac{\epsilon-1}{\epsilon}}} M_t \tag{22}$$

$$C_{it}(z) = \frac{z^{\frac{\epsilon-1}{\epsilon}}}{\mathcal{Z}^{\frac{\epsilon-1}{\epsilon}}} Y_t \qquad p_{it}(z) = \left(\frac{z}{\mathcal{Z}} \right)^{-\frac{1}{\epsilon}} P_t \tag{23}$$

for an arbitrary $z \in [\underline{z}, \overline{z}]$.

Corollary 1.2 shows that changes in the money supply are fully absorbed into prices, making output constant across fundamental equilibria[8]. Additionally, $p_{it}(z)$ and P_t are a function of the money supply at the stationary equilibrium. When convenient, I explicitly denote this dependence, by writing $p_i(z, M_t)$ and $P(M_t)$. Furthermore, for an arbitrary $z \in [\underline{z}, \overline{z}]$, $\theta_t(z), m_{it}(z), C_{it}(z)$ and $p_{it}(z)$ are rescaled versions of their average counterparts, where the rescaling factor depends only on z. Thus, these stationary equilibria reflect fundamentals as more productive entrepreneurs have higher revenues, monetary holdings and consumption, and lower prices than the average.

[7]The tractability arises mainly due to the elimination of difficulties related to potential non-homothety. However, the property that income and substitution effects cancel out also helps in the analytical proofs but does not seem essential for obtaining the results, as is shown in Subsection 4.2.

[8]This feature does not rely on any specific functional form of $u(\cdot)$.

3.1.3 Immediate return to the stationary equilibrium

In this subsection, I study under what circumstances high-cash agents fully deplete their money at $t = 1$. In this case, since $s_{it}(z) = 0$ for all $z \in [\underline{z}, \bar{z}]$ and $i \in \{c, u\}$, high-cash agents at $t = 2$ hold only the money earned from selling their products at $t = 1$, like their low-cash counterparts. By (13), equally productive high- and low-cash agents set the same price, earning $R_1(z) = (1 + \tau^A)m_0(z)$. Therefore, individual and aggregate prices match those in the helicopter drops case, and output remains $Y_t = Y_0$.

Notice that, by the buyer's first order condition, full depletion happens when:

$$\frac{1}{\beta}\left(1+\tau^A\right) m_0(z) = \frac{1}{\beta} P_2 C_{h2} \geq P_1 C_{h1} = m_{h1}, \tag{24}$$

which implies that high-cash agents fully deplete their money if:

$$|\tau| = \tau \leq \frac{1-\beta}{\beta - \eta \frac{M_{c0}}{M_0}} \qquad \text{if } \tau > 0 \tag{25}$$

$$|\tau| = -\tau \leq \frac{1-\beta}{\eta \frac{M_{c0}}{M_0}} \qquad \text{if } \tau < 0 \tag{26}$$

This means that the high-cash agents fully deplete their money holdings if, and only if, the monetary shock is "low enough". Notice that (25) is only defined for $\eta < \beta \frac{M_0}{M_{co}}$. To understand why, notice that, if $\eta \in \left[\beta \frac{M_0}{M_{c0}}, 1\right]$, then $\frac{1}{\beta}\left(1+\tau^A\right) m_0(z) > (1+\tau) m_0$, meaning that the full depletion condition is satisfied for any $\tau \neq 0$ and for all $z \in [\underline{z}, \bar{z}]$. Intuitively, when the connected agent fully depletes their cash holdings, the resulting drop in monetary holdings from one period to the next is too small to incentivize saving. Thus, henceforth, I assume the most interesting case: $\eta \in \left(0, \beta \frac{M_0}{M_{co}}\right)$.

When (24) is satisfied, the economy is at the new equilibrium from period $t = 2$ onwards. However, there are important distributional effects at the period $t = 1$. High-cash and low-cash agents' consumption is given, respectively, by:

$$C_{i1}(z) = C_0(z)\left(\frac{m_{i1}(z)}{(1+\tau^A) m_0(z)}\right) \quad fori \in \{c, u\}.$$

with $C_{h1}(z) > C_0(z) > C_{l1}(z)$. Now, I examine the case in which (24) does not hold.

3.1.4 Gradual return to the stationary equilibrium

When the shock is large enough — that is, (24) is not satisfied, — high-cash agents smooth their consumption. Then, prices and output do not go to their equilibrium values immediately anymore. Notice that, for as long as high-cash agents do not fully deplete their resources, we must have $M_t^C < (1+\tau^A)M_0$. Evidently, for any $t \in \{1, 2, ...\}$, $M_t^C > M_0$ for $\tau > 0$ and $M_t^C < M_0$ for $\tau < 0$. In the following proposition, I fully characterize the transition dynamics after the shock. Most of these findings will be shown to generalize to a setup with CRRA utility function in Subsection 4.2.

Proposition 3. *Let $T^{MAX} < \infty$ be defined as:*

$$T^{MAX} := \arg\max_{T\in\mathbb{N}} \left[\beta^{T-1} > \frac{1+\tau^A}{1+\mathbb{1}_{\tau>0}\tau}\right] \tag{27}$$

Under log-utility, the moment where prices achieve their new stationary equilibrium level satisfies $T \leq T^{MAX}$. Moreover, if we define:

$$\overline{X_{i1}} := X_{i1}(\mathcal{Z}_i) \qquad \text{for } X \in \{C, \theta, m\} \text{ and } i \in \{h, l\} \tag{28}$$

as the average level of consumption, relative revenues, and monetary holdings among either high- or low-cash agents, we have:

$$X_{i1}(z) = \overline{X_{i1}} \frac{z^{\frac{\epsilon-1}{\epsilon}}}{\mathcal{Z}_i^{\frac{\epsilon-1}{\epsilon}}} \qquad \text{for } X \in \{C, \theta, m\} \text{ and } i \in \{h, l\} \tag{29}$$

for all $t = \{0, 1, ...\}$ due to the homothety of log-utility. Moreover, the following results hold for $t = \{1, ..., T-1\}$ and for any arbitrary $z \in [\underline{z}, \overline{z}]$:

$$\begin{array}{lll}
(a) & \frac{m_{ht}(z)}{P_t} > C_{ht}(z) > C_0(z) > C_{lt}(z) = \frac{m_{lt}(z)}{P_t} & \\
(b) & C_{h,t+1}(z) < C_{ht}(z) \qquad \text{and} \qquad m_{h,t+1}(z) < m_{ht}(z) & \\
(c) & C_{l,t+1}(z) > C_{lt}(z) \qquad \text{and} \qquad m_{l,t+1}(z) > m_{lt}(z) & \\
(d) & p_{ht}(z) > p^H(z, M_t) > p_{lt}(z) > p^H(z, M_t^C) & \\
(e) & P_t > P^H(M_t^C) & \\
(f) & \theta_0(z) > \theta_{h,t+1}(z) > \theta_{ht}(z) & \\
(g) & \theta_0(z) < \theta_{l,t+1}(z) < \theta_{lt}(z) \qquad \text{and} \qquad p_{l,t+1}(z) > p_{lt}(z) & \\
(h) & h_{l,t+1}(z) = h_{lt}(z) = h_{lt}(z') = h_{l,t+1}(z') \qquad for every z, z' \in [\underline{z}, \overline{z}] & \\
(i) & Y_0 \geq Y_{t+1} > Y_t \qquad \text{with strict inequality for } t = \{1, ..., T-1\}. &
\end{array}$$

Proposition 3 shows that the number of periods needed for reaching the new equilibrium satisfies (27). To build intuition, notice that high-cash agents' money holdings fall between $t = 1$, when high-cash agents own $m_{h1}(z) = \theta_0(z)(1 + \mathbb{1}_{\tau>0}\tau)M_0$, and $t = T + 1$, when they own $m_{h,T+1}(z) = \theta_0(z)(1 + \tau^A)M_0$. For a fixed individual shock τ, the size of this fall depends on how big the aggregate shock, τ^A, is, as it affects the demand agents will face. If the aggregate shock is small, high-cash agents' revenues are lower, encouraging them to keep savings for longer to smooth consumption.

Unlike the case of an immediate return to the stationary equilibrium, now, *money is not neutral in the aggregate*. The proposition above shows that GDP *falls* under log-utility after the shock — for contractionary or expansionist shocks alike — as the price level remains above the helicopter drops level compatible with the amount of money in circulation throughout the transition period, *i.e.*, $P_t > P^H(M_t^C)$ for $t < T$. This means that the aggregate price is higher than the one that ensures money neutrality.

For an expansionist shock, *i.e.*, $\tau > 0$, unconnected agents gradually increase their prices along the transition path. This is because the future value of holding money for them, given by $\beta / m_{I,t+1}(z)$, is inversely proportional to their future monetary holdings. As they get richer, the value of money decreases, allowing their prices to approach their final, higher level. Since their "market share" falls over time, their rising monetary wealth stems from a higher amount of money in circulation, M_t^C. Finally, note that even unconnected agents set prices above $p^H(z, M_t^C)$, meaning that their prices remain *too high* to sustain output at Y_0.

On the other hand, for a contractionary shock (*i.e.*, $\tau < 0$), the price of the goods chosen by connected agents *undershoots*, since connected agents are made poorer, and they need to decrease their price on impact more than the unconnected to remain competitive. They gradually increase their prices as their monetary holdings grow. They also work more than the unconnected for $t \in \{1, ..., T-1\}$. To study the quantitative implications of the model, I now conduct a simulation.

3.1.5 Simulation

For the simulation, I assume a uniform productivity distribution for simplicity. I also do away with the common support assumption to facilitate calibration[9]. I fix the lower bound for the productivity of both types of agent at $\underline{z}_c = \underline{z}_c = 0.2$ and calibrate the upper bounds as will be described in more detail below. I normalize the initial monetary base and aggregate prices to $M_0 = 1$ and $P_0 = 1$, and aggregator output is also normalized to $Y_0 = 1$ as a consequence. Table 1 summarizes the calibration.

As usual, I set the elasticity of substitution to $\epsilon = 11$ to get a 10% markup. Moreover, each period is assumed to be a quarter, and, thus, I set the rate of time discount to $\beta = 0.99$ to get a 1% quarterly real interest rate in equilibrium. I also set the shock size to $\tau = 0.2$, which amounts to connected agents becoming 20% richer than the unconnected with the same productivity. In our calibration, this leads to, roughly, a 11%

[9] This assumption is useful — though not essential — for the proofs. However, due to the homotheticity of the utility function, the relaxation of this assumption is inconsequential. Alternatively, we can assume that $F_u(z) = 1$ for $z \in [\bar{z}_u, \bar{z}_c]$ without having to relax it.

Parameter	Definition	Value
ϵ	Elasticity of substitution	11
β	Rate of time discount	0.99
γ	Labor disutility	8.1
η	Fraction of connected agents	0.27
M_0	Initial money supply	1
τ	Individual monetary shock	0.2
$\underline{z}_c$	Minimum productivity among connected	0.2
$\underline{z}_u$	Minimum productivity among unconnected	0.2
$\overline{z}_c$	Maximum productivity among connected	13.311
$\overline{z}_u$	Maximum productivity among unconnected	3.3176

Table 1: Parameter values for the simulation

aggregate shock. Although this is fairly large, 1) the shock is not persistent, and 2) as can be seen in Table 3 (see C), since 1999, shocks to the monetary base of at least 10% occurred in a total of nine quarters, eight of which were positive shocks.

I use data from the Survey of Consumer Finances (SCF) to retrieve the fraction of connected agents in the U.S. economy. The SCF contains information U.S. households' financial asset ownership. Following the limited participation literature[10], I classify households as connected if they own stocks or bonds, directly or through mutual funds. I disregard, whenever possible, foreign bonds, since, by our definition, connected agents should get richer with interest rate cuts by the Fed. I also disregard retirement funds and US savings bonds, due to illiquidity and low relevance.

In 2022, 27% of U.S. households met this definition, with an average income 2.06 times the national average[11]. I therefore, set $\eta = 0.27$ and calibrate $\overline{z}_c$, $\overline{z}_u$ and γ to match: 1) the normalizations described above, 2) a stationary equilibrium labor supply of $1/3$[12], corresponding to an 8-hour workday, and 3) a ratio of the monetary wealth of connected agents and the whole population of $M_{c0}/M_0 = 2.06$. These targets are achieved exactly by construction. By setting $\underline{z}_c = \underline{z}_u = 0.2$, we also obtain a ratio of the income of the 90^{th} to the 10^{th} percentiles of approximately 11.2149, which is close to the actual ratio, according to the SCF, of $10,4762$.

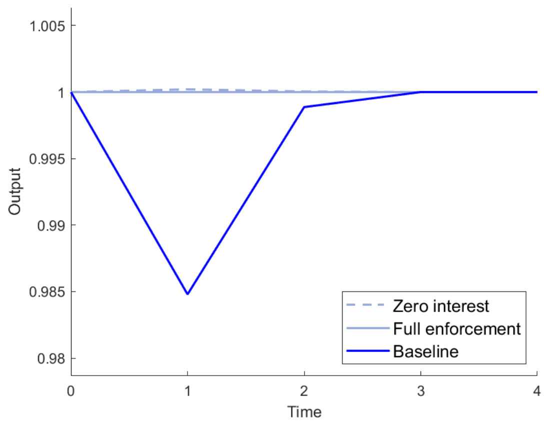
[10] Mankiw and Zeldes (1991) and Vissing-Jørgensen (2002) show that stock owners' consumption is more sensitive to stock market excess returns, aligning with our definition of the connected agents.

[11] I consider households' reported revenues in a normal year.

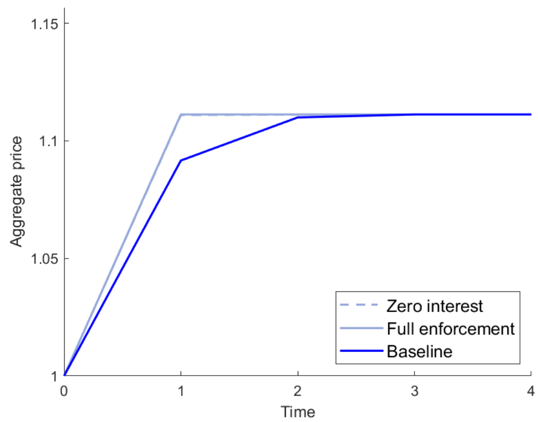
[12] In the model, for simplicity, I do not assume any upper bound to the labor of the entrepreneur. Nevertheless, for the simulation, I set the labor endowment to 1. This does not affect the solutions for as long as the labor constraint does not bind.

I compute Gini indexes for consumption, revenues, and monetary holdings. The stationary equilibrium income Gini obtained is $Gini_0 \approx 0.4363$. This is quite close to the actual one, which has fluctuated around 0.4 since the 1990s. I also compute the total factor productivity (TFP) in the simulations. The idea behind the TFP measure is to see how productive a representative household would need to be to produce the aggregate output given the average amount of labor in the economy $\overline{h}_t = \sum_{i\in\{c,u\}} \eta_i \int_{\underline{z}}^{\overline{z}} l_{it}(z) dF_i(z)$. Since the technology is linear, this means that $TFP_t := Y_t/\overline{h}_t$.

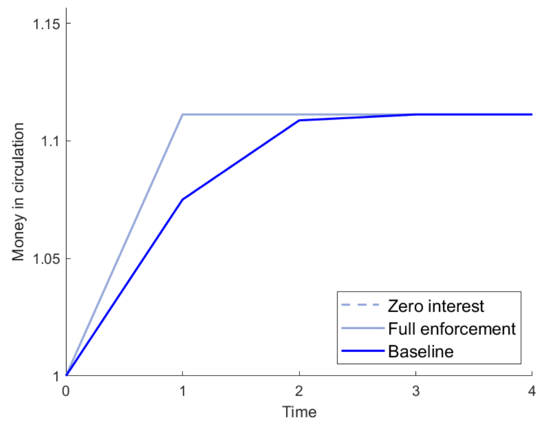
Figure 1 and Figure 2 show, respectively, the paths for aggregate and individual level variables under the baseline economy, the economy with full enforcement of bond contracts — discussed in Subsection 3.2, — and the zero interest rate economy — discussed in Subsection 3.3. Figure 2 considers connected and unconnected agents with productivity $z_c = z_u = \mathcal{Z}$. Since the logarithmic utility is homothetic, these graphs are identical for other productivity levels up to a re-scaling.



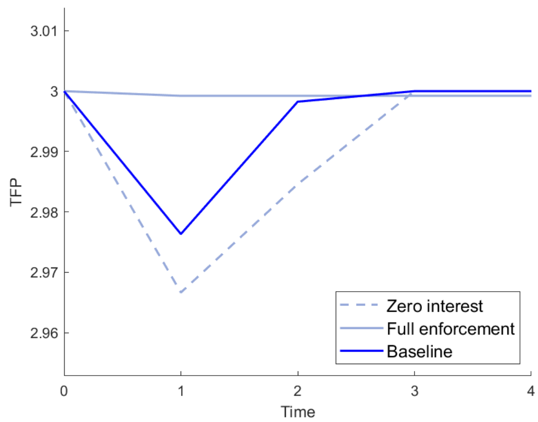
(a) Path of output, Y_t



(b) Aggregate price, P_t



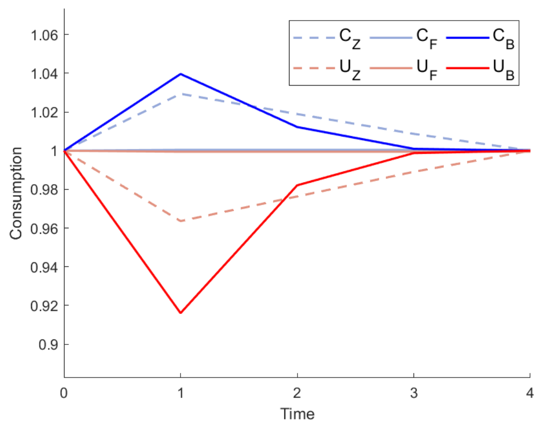
(c) Money in circulation, M_t^C



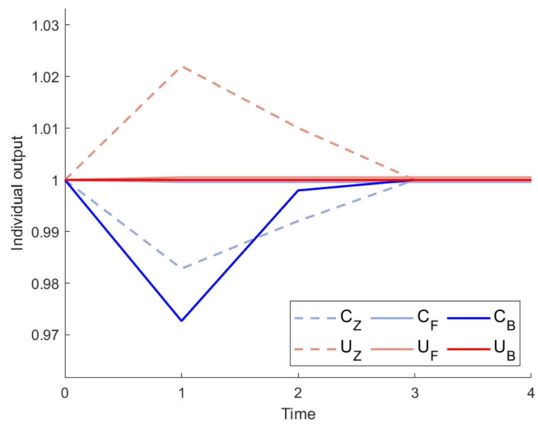
(d) Total Factor Productivity

Figure 1: Paths for aggregate variables in the three economies

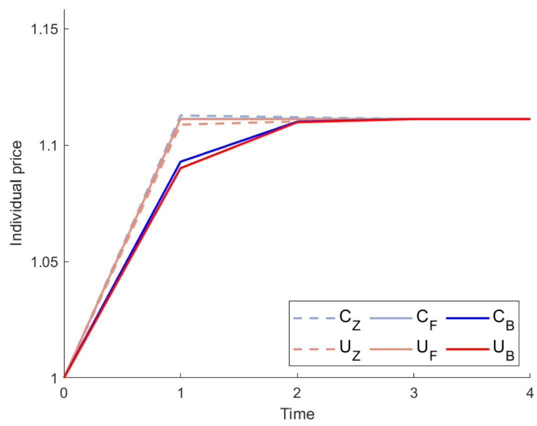
In our calibration, the transition path lasts three quarters. Figure 1a indicates that output falls roughly 1.5% on impact, and nearly recovers at $t = 2$. This fall is concentrated on the production of connected agents, as can be seen in Figure 2a. This is because, as shown in Proposition 3, labor and output of unconnected agents are constant because they set the price to keep the demand they face unchanged. Besides, since connected agents are more efficient on average, allocative efficiency goes down with the shock — as can be seen in Figure 1d. This means that monetary shocks can generate fluctuations in TFP if they favor more/less productive agents.



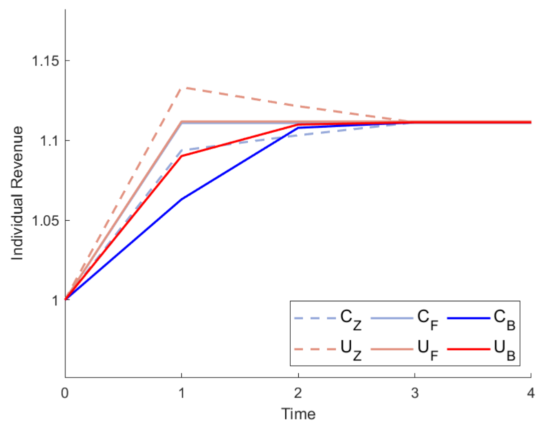
(a) Individual consumption, $C_{it}(\mathcal{Z})$



(b) Individual output, $y_{it}(\mathcal{Z})$



(c) Individual prices, $p_{it}(\mathcal{Z})$



(d) Individual revenues, $R_{it}(\mathcal{Z})$

Figure 2: Paths for individual variables in the three economies

Notes: Connected (C) and unconnected (U) cases. Subscripts denote: baseline (B), full enforcement (F), and zero interest rate (Z).

Figure 2a shows that connected agents' consumption rises by roughly 3.96% on impact, while unconnected agents' consumption drops by about 8.39% due to rising prices. Moreover, Figure 1c shows that 67.46% of the injected money is put in circulation at $t = 1$. According to Figure 2d, connected agents' revenues grow less than that of the unconnected due to the reduction in their output. So, the fact that their income falls at $t = 1$ makes them save around $0.3254\tau m_0(\mathcal{Z})$ to afford $P_2 C_{c2}(\mathcal{Z}) = \beta P_1 C_{c1}(\mathcal{Z})$.

All these patterns are reflected in the Gini indexes, as can be seen in Figure 3, where I use the following notation: I denote each Gini index as G_k, with $G \in \{C, R, M\}$ corresponding, respectively, to the consumption, revenue, and monetary holdings Gini; and $k \in \{B, F, Z\}$ corresponding, respectively, to the baseline, full enforcement and zero interest rate economies. Notice that the Gini for monetary wealth is the one that goes up the most, indicating a big increment in wealth inequality, as the already relatively rich connected agents become richer with the shock.

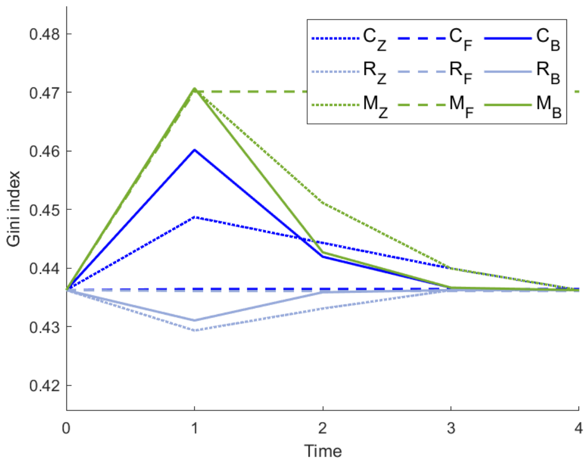


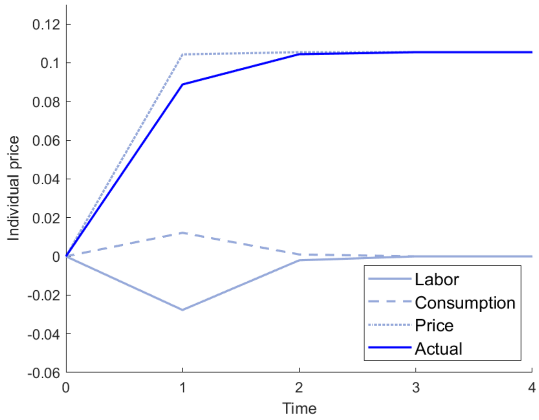
Figure 3: Gini indexes in the three economies

Notes: $G \in \{C, R, M\}$ stand for consumption, revenue, and money holdings; $k \in \{B, F, Z\}$, for baseline, full enforcement, and zero interest rate.

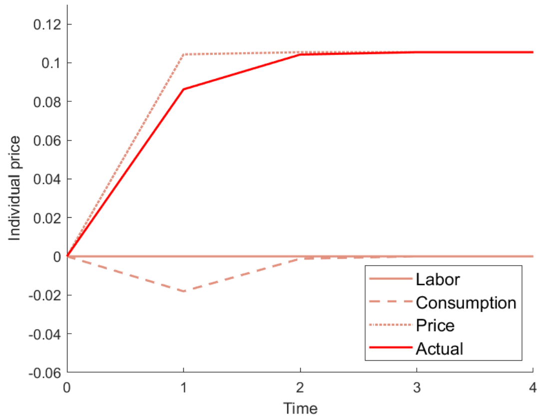
The Gini for income falls, reflecting the mechanism that re-establishes the equilibrium since the unconnected agents choose low prices to recover their real money balances. Thus, consumption inequality grows, but less than wealth inequality. Concerning the aggregate price, Figure 1b indicates that roughly 82.4% of the increment in the aggregate prices happens already at the first period after the shock. This means the model produces endogenous aggregate price stickiness since money is gradually put into circulation due to the consumption-smoothing behavior of connected agents[13].

As for individual prices, Figure 2c shows that the differences between connected and unconnected agents are small. Hence, the bulk of the disparities in revenue is due to the adjustment on the labor margin. To better understand this, I use (13) and

[13]Elsewhere, it was shown that the aggregate price grows *more* than what should be the case, given the amount of money put in circulation, that is, $P_t > P^H(M_t^C)$ for the periods $t = 1, ..., T$. Nevertheless, the stickiness comes from the fact that $P_t < P^H((1 + \tau^A)M_0)$ for $t = 1, ..., T$, that is, the price is *lower* than the final level it attains when the whole money supply is in circulation.



(a) Connected agents' prices, $\log(p_{ct}(\mathcal{Z}))$



(b) Unconnected agents' prices, $\log(p_{ut}(\mathcal{Z}))$

Figure 4: Decomposition of individual prices in the baseline economy

decompose the price series into three components: labor, future consumption, and future aggregate price. I take the logarithm of connected and unconnected agents' prices and their components, then normalize all series by subtracting the logarithm of their initial stationary equilibrium values. This ensures that the logarithm of the components sums to the logarithm of prices. I plot the results on Figure 4.

The graphs show that, for connected agents, the primary driver of price sluggishness is the decline in labor, which reduces its marginal disutility and, consequently, marginal costs. However, their higher consumption partially offsets this effect. For unconnected agents, the fall in consumption is the main factor pushing prices down, which is necessary to stay competitive and replenish purchasing power. The expectation of high future prices is the main reason for the relatively strong initial price response. Since part of the dynamics described in this section stems from idle money balances, the next section introduces borrowing by reinstating the bonds market.

3.2 Full Enforcement

Now, I consider the case of full enforcement of debt contracts, where bond sales are subject to a borrowing limit, namely, $l_t(z, m_{it}(z)) = m^{-}_{i,t+1}(z) = R_{it}(z)$. Intuitively, since the repayment of one's debt is always enforced, the entrepreneur must only have enough cash at the beginning of $t+1$ to repay their debt. This means that for $i \in \{h, l\}$:

$$q_t = \beta \frac{P_t C_{it}(z)}{P_{t+1} C_{i,t+1}(z)}$$

In the fundamental stationary equilibrium, we must have $q_0 = \beta$, since $P_t C_{it}(z) = P_{t+1} C_{i,t+1}(z)$, and $b_0(z) = 0$ for all individuals with $z \in [\underline{z}, \overline{z}]$ and $i \in \{c, u\}$.

3.2.1 Helicopter drops of money

Given that the helicopter drops case does not distort relative money balances, the dynamics is identical to the case where bonds are absent. Since equally productive agents have identical money holdings after the shock, there is no role for borrowing. Next, I analyze the situation where there is an uneven monetary injection.

3.2.2 Uneven access to the new money and full enforcement of bond contracts

As before, I assume that connected agents are the first to see their monetary holdings change. Then, the following corollary to Proposition 1 holds.

Corollary 1.3. *For any $\tau \neq 0$, if there is full enforcement of bond contracts, the economy goes immediately to the new stationary equilibrium at $t = 1$. The equilibrium bond price is $q_t = \beta$ for $t = 1, 2, \ldots$. Moreover, we have:*

- $C_{it}(z) = C_{i,t+1}(z)$, $m_{it}(z) = m_{i,t+1}(z)$ *and* $b_{it}(z) = b_{i,t+1}(z)$ *for* $i \in \{c, u\}$ *and* $z \in [\underline{z}, \overline{z}]$;
- $C_{ht}(z) > C_{lt}(z)$, $p_{ht}(z) > p^H(z, (1+\tau^A)M_0) > p_{lt}(z)$ *and* $R_{lt}(z) > \theta_0(z)M_t > R_{ht}(z)$;

Expenditures with consumption is given by:

$$P_t C_{ct}(z) = (1-\beta)(1+\tau)m_0(z) + \beta R_{ct}(z) \tag{30}$$

$$P_t C_{ut}(z) = (1-\beta)m_0(z) + \beta R_{ut}(z) \tag{31}$$

Let $\overline{\mathcal{R}}(z) := (1-\beta)\tau m_0(z)/\beta$. Then, there is an upper and a lower bound to the difference in revenues between connected and unconnected agents:

$$\overline{\mathcal{R}}(z) > R_{ut}(z) - R_{ct}(z) > 0 \qquad \text{if } \tau > 0 \tag{32}$$

$$\overline{\mathcal{R}}(z) < R_{ut}(z) - R_{ct}(z) < 0 \qquad \text{if } \tau < 0 \tag{33}$$

and for the difference in consumption expenditures between them:

$$\beta\overline{\mathcal{R}}(z) > P_t C_{ct}(z) - P_t C_{ut}(z) = \beta\left[\overline{\mathcal{R}}(z) - \beta(R_{ut(z)} - R_{ct}(z))\right] > 0 \quad \text{if } \tau > 0, \tag{34}$$

$$\beta\overline{\mathcal{R}}(z) < P_t C_{ct}(z) - P_t C_{ut}(z) = \beta\left[\overline{\mathcal{R}}(z) - \beta(R_{ut(z)} - R_{ct}(z))\right] < 0 \quad \text{if } \tau < 0. \tag{35}$$

According to the corollary above, the economy immediately reaches the new stationary equilibrium, with persistent consumption differences between connected and unconnected agents. Money holdings remain permanently fixed at their post-shock levels, indicating hysteresis. The prospect of future interest payments allows high-cash agents to set higher prices, work less, and accept lower current revenue while maintaining a higher consumption standard indefinitely. Corollary 1.2 also implies that $|R_{ut}(\mathcal{Z}) - R_{ct}(\mathcal{Z})| < 0.002$ and $|P_tC_{ct}(\mathcal{Z}) - P_tC_{ut}(\mathcal{Z})| < 0.002$. For comparison, in the bondless economy, at $t = 1$, these gaps were $|R_{u1}(\mathcal{Z}) - R_{c1}(\mathcal{Z})| = 0.0271$ and $|P_1C_{c1}(\mathcal{Z}) - P_1C_{u1}(\mathcal{Z})| = 0.1349$. As expected, the introduction of bonds significantly reduces heterogeneity between agents but makes these smaller differences permanent.

Figure 1a shows that aggregate output is unaffected by the shock, suggesting that, with perfect enforcement of bond contracts, monetary policy is neutral at the aggregate level. Similarly, Figure 1b shows that the aggregate price behaves as in the helicopter drops economy. Therefore, in this setup, a representative agent model would provide a reasonable approximation of the economy's aggregate behavior. As shown in Figure 2a and Figure 2c, the gaps in consumption and prices are almost unnoticeable. However, differences in revenues and output are more perceptible. Unlike before, unconnected agents now produce slightly *more* than in the initial stationary equilibrium, though this increment is quantitatively negligible.

It is easy to show that the output produced by the unconnected agents is:

$$y_{ut}(z) = \underbrace{z\left(\frac{\epsilon-1}{\epsilon}\right)\frac{\beta}{\gamma}}_{y_0(z)} \underbrace{-(1-\beta)\frac{b_{ut}(z)}{p_{ut}(z)}}_{>0},$$

where the first term is the output needed to balance the trade-off between current labor and future consumption without interest payments. The increment corresponds to the additional output that must be produced to cover interest payments on bonds. In fact, Corollary 1.2 implies that $P_tC_{ut}(z) < R_{ut}(z)$, with the difference precisely accounting for coupon payments. Hence, monetary policy remains neutral in the aggregate in the presence of a bonds market due to 1) a smaller increment in connected agents' consumption off-setting part of the fall in output by increasing the value of future consumption, and 2) the increment in production by unconnected agents to pay interests.

3.3 Exogenously Set Interest Rate

Now, I exogenously set the interest rate at either $\beta < q_t \leq 1$ or $q_t < \beta$. This is relevant because the central bank directly determines it. For $q_t > \beta$ ($q_t < \beta$), an excess supply (demand) of bonds arises, restricting bond sales to the available demand (supply). In the case of $\beta < q_t \leq 1$, I assume:

$$l_t(z, m_{lt}(z)) = \frac{\theta_{lt}(z)}{\theta_{lt}(\mathcal{Z}_l)}\left(\frac{\eta_h}{1-\eta_h}\int_{\underline{z}}^{\bar{z}} b_{h,t+1}(z)dF_h(z)\right). \tag{36}$$

This means that each low-cash agent can obtain a fraction of total bonds proportional to their "market share" relative to the average low-cash agent. When $q_t < \beta$, there is excess demand for bonds. Thus, I add a rationing constraint, $b_{h,t+1}(z) \leq r_t(z, m_{ht}(z))$, with $r_t(z, m_{ht}(z))$ defined similarly to (36). Since equilibrium in the bonds market requires $q_t = \beta$, I will assume that q_t remains fixed at the exogenously set level during the transition to ensure the exercise is meaningful. Proposition 4 guarantees that the economy should eventually return to the fundamental stationary equilibrium.

Proposition 4. *If there is a constant $\beta < q_t \leq 1$ or $q_t < \beta$, then there must be a period $T < \infty$ at which the economy returns to the fundamental stationary equilibrium.*

The intuition is as follows: if the interest rate is below its equilibrium value, connected agents are not compensated enough for deferring consumption. As a result, their optimal consumption path features declining consumption during the transition, leading to decreasing monetary holdings. Eventually, these holdings will be so close to their stationary equilibrium level that these agents are better off spending it all at once, as in the bondless economy. If the interest rate exceeds its equilibrium level, unconnected agents avoid borrowing indefinitely, as it is very costly to do so.

To explore a middle ground between the two previous cases, I focus on the scenario where $q_t = 1$. In this case, there are no interest payments to connected agents, but their idle cash balances are still channeled to the unconnected. Then, Figure 1b shows that the aggregate price exhibits nearly zero stickiness. As shown in Figure 6 in B, the labor and consumption components of prices cancel out for both connected and unconnected agents. This occurs because unconnected agents face higher labor disutility, raising their marginal cost as they must work harder to amortize their debt. This offsets their incentives to set lower prices due to lower consumption. Consequently, $Y_t = M_t^C / P_t$ is also nearly unaffected, increasing negligibly.

Figure 2a indicates that the connected agents' consumption grows less than in the baseline economy due to higher prices. The consumption gap relative to unconnected agents is smaller than in the baseline economy but remains larger than in the full enforcement case. Figure 2b shows that the output and revenues for all agents exceed baseline levels, with unconnected agents' revenue overshooting. However, in B, Figure 7 suggests that this overshooting is insufficient to make the monetary holdings of connected and unconnected agents converge as fast as in the baseline economy, due to debt amortization.

Overall, the aggregate variables, except for TFP, are compatible with a representative agent model, but the differences between connected and unconnected agents remain significant, which is reflected in the Gini indexes. Still, Figure 3 shows that, although the Gini for monetary wealth and income do not change by very much relative to the baseline, the consumption Gini increases by much less now, which illustrates the role played by bonds in allowing for risk sharing.

3.4 Welfare

I now analyze the welfare implications of the models discussed so far. I adopt a utilitarian specification of the welfare function and give all individuals equal weight. The function is, then, given by:

$$W_t = \sum_{s=t}^{\infty} \beta^{s-1} \sum_{i \in \{c,u\}} \eta_i \int_{\underline{z}}^{\overline{z}} \left(log(C_{is}(z)) - \gamma \frac{h_{is}(z)^{1+\zeta}}{1+\zeta} \right) dF_i(z)$$

Now I define the short-run consumption equivalent as follows:

$$W_t = log(\Phi C_0(z)) - \gamma \frac{h_0(z)^{1+\zeta}}{1+\zeta} + \frac{\beta}{1-\beta} W_0,$$

where $\Phi - 1$ is the uniform increment/decrement to the fundamental stationary equilibrium consumption of all agents *in the first period* that would yield the same welfare level as the actual allocation. This measure is used for the sake of readability, since, due to the transitory nature of the shock, it does not matter in the long run. As a result, the usual consumption equivalent measure in terms of lifetime consumption is two orders of magnitude smaller.

Table 2 shows the results. C contains tables for some counterfactual exercises aimed at better understanding what drives the welfare differences. I have included the cases for $M_{c0}/M_0 \in \{2.06, 1, 0.6\}$ for ease of comparison. The individual level shocks are kept at $\tau = 0.2$ throughout, and the aggregate shock is $\tau^A = 0.1112$. Hence, I let the value of the fraction of connected agents be, respectively, $\eta \in \{0.27, 0.5562, 0.927\}$ to ensure that the aggregate shock is constant[14].

Model	$M_{c0}/M_0 = 2.06$	$M_{c0}/M_0 = 1$	$M_{c0}/M_0 = 0.6$
Baseline	-5.5304%	-0.4016%	6.6595%
Full enforcement	-5.0228%	-0.0021%	6.8995%
Zero interest rate	-5.0894%	-0.1006%	6.755%

Table 2: Welfare analysis

The table shows that, under the calibration adopted above, the average entrepreneur would be just as well off accepting a one-period fall of 5.53% in their consumption

[14]Subsection 4.7 will show that changing η and M_{c0}/M_0 produces the same aggregate and individual level paths if both aggregate and idiosyncratic shocks are maintained.

as in the baseline economy. Notably, welfare falls with the monetary shock when $M_{c0}/M_0 = 2.06$ and $M_{c0}/M_0 = 1$, but increases when $M_{c0}/M_0 = 0.6$. This occurs because, in the latter scenario, connected agents are, on average, poorer. Consequently, a positive monetary shock reduces inequality and boosts consumption for agents who, on average, derive a higher marginal utility from consumption. Among the shocked economies, the baseline one exhibits the lowest welfare.

The full enforcement scenario yields the highest welfare post-shock. Although the improvement is modest — approximately 0.5% under our calibration, — it nearly undoes the decline caused by the shock when $M_{c0}/M_0 = 1$. The zero interest rate model represents a middle ground between the two limiting cases but it is closer to the full enforcement scenario. This indicates that: 1) financial frictions reduce welfare after a monetary shock, and 2) an exogenously low interest rate reduces welfare relative to the equilibrium — *natural* — rate.

Moreover, the table also shows that monetary policy has two types of distributional effects. On the one hand, it unequivocally *decreases* welfare by increasing inequality between connected and unconnected agents with the same productivity. On the other hand, it may primarily benefit agents with either higher or lower average productivity, leading to an ambiguous overall effect on inequality — depending on how well-off connected agents are relative to the unconnected.

Table 4, in C, shows that the main factor lowering welfare in the baseline economy is the increment in inequality for $M_{c0}/M_0 \in \{2.06, 1\}$, though the fall in output also plays a minor role. Table 5 presents counterfactuals for the full enforcement economy. Enforcing the attainment of the same equilibrium as the other economies after T periods results in minimal welfare loss. This suggests that permanent non-fundamental inequality is the main driver of the welfare decline under full enforcement.

Finally, three exercises regarding the zero interest rate situation are shown in Table 6, in C: (1) I impose the same output decline as in the baseline economy while maintaining the inequality level generated by the zero-interest-rate model; (2) I impose the inequality level in the baseline model but retain the rise in output under the zero interest rate regime; (3) I eliminate non-fundamental inequality. Compared to the baseline, the increase in welfare under the zero interest rate model is primarily driven by the reduction in the disequalizing effects of the monetary shock when $M_{c0}/M_0 = 2.06$, where it benefits relatively richer agents. For $M_{c0}/M_0 = 0.6$, welfare improves *despite* the reduced redistribution induced by the shock — which benefits poorer agents — because it reduces non-fundamental inequality.

4 Sensitivity Analysis

In this section, I assess how my findings respond to changes in assumptions and parameters. I begin by analyzing a negative monetary shock. Secondly, I adopt a CRRA utility specification. Third, I allow the Frisch elasticity of labor supply to vary. I then modify the size of the individual shocks, while keeping the aggregate shock constant, and interpret the findings in the context of futures markets. Additionally, I fix individual and aggregate shock sizes but vary the amount of connected agents and their relative wealth. Next, I examine how the length of the transition path varies with the fraction of connected agents. Lastly, I study collateral constraints. I concentrate on the baseline economy, as it provides a clear picture of how the patterns evolve. Most graphs can be found in Appendix B, and Table 9 summarizes the results.

4.1 A Negative Shock

I impose an equivalent negative shock of $\tau = -0.1667$, which also makes the high-cash agents 20% richer than their low-cash counterparts while keeping all other parameters unchanged. Figure 8 in Appendix B plots the graphs for the aggregate and individual variables. The patterns resemble those observed in the previous model, but now the roles of connected and unconnected agents are reversed. As a result, the Gini coefficients and TFP go in the opposite direction. Interestingly, the aggregate price *undershoots*, as shown in Figure 8b, due to idle money balances.

4.2 CRRA Utility Specification

I now assume that the utility function is given by:

$$u(c) = \frac{C^{1-\alpha}}{1-\alpha},$$

and consider $\alpha \in \{0.5, 2\}$, corresponding to an intertemporal elasticity of substitution (IES) of $IES_{\alpha=0.5} = 2$ and $IES_{\alpha=2} = 0.5$. I recalibrate the model and assume, for simplicity, that connected and unconnected agents' average productivity is the same. The model's fundamental patterns remain unchanged. Figure 9a shows that, under a lower IES, output declines more due to stronger consumption smoothing. Figure 9 indicates that prices and revenues respond more sharply, as prices become more sensitive to changes in consumption for $IES_{\alpha=2} = 0.5$. Thus, despite connected agents' preference for extended consumption smoothing, higher prices shorten the transition.

4.3 Varying Frisch Elasticity Of Labor Supply

I now let the inverse Frisch elasticity of labor supply vary. I consider four cases: $\zeta \in \{0, 0.5, 1, 2\}$, which correspond, respectively, to 1) an infinite Frisch elasticity, 2) a "macro" elasticity of 2, 3) the baseline value, and 4) a "micro" elasticity of 0.5. Figure 10 shows that there is close to no difference across cases in the length of the transition path. However, for large Frisch elasticities, there is a bigger initial response in prices, a larger fall in output, and more distortions in allocative efficiency. This is due to a higher responsiveness in connected agents' output and, thus, revenues.[15]

4.4 Varying Access To Financial Markets

I vary the fraction of connected households, η, while fixing the aggregate shock, τ^A. This can be seen as changing access to financial markets. To better control parameter variations, I assume no differences in average productivity between connected and unconnected, *i.e.*, $M_{c0}/M_0 = 1$. I consider $\eta \in \{0.1, 0.27, 0.5, 0.75\}$, which corresponds to the individual level shocks $\tau \in \{1.1124, 0.412, 0.2225, 0.1483\}$[16]. Figure 11 shows faster transition and price adjustment for higher η. Output declines the least when η is the highest and the lowest. For $\eta = 0.1$, the combination of slow transition and sluggish prices produces a lower fall in output relative to the other cases.

The heterogeneity in outcomes is highest when η is low, due to a higher individual monetary shock. This is reflected in the Gini indexes. Besides, a slight fall in TFP, due to the diminishing returns introduced by the CES aggregator, is visible in Figure 11d. Finally, Table 7 shows that welfare falls the least in the economies with higher η across all model specifications. Overall, the monetary shock is less distortionary when access to financial markets is more widespread, despite exposing more agents to monetary policy risk, as it makes the shocks work more like helicopter drops.

[15] Interestingly, Figure 10h shows that, under $\zeta = 0$, we observe the same pattern as in Williamson (2008): the price chosen by connected agents overshoots, while the price of unconnected agents' products grows slowly. In his model, demand in the goods market of connected agents jumps on impact in nominal terms, and the new money gradually flows into the unconnected goods market due to imperfect market segmentation. In my framework, under a perfectly elastic labor supply, marginal costs are fixed, meaning that individual prices become proportional to future consumption expenditure.

[16] This negative relationship between market thickness and the impact of individual shocks is well-documented in the literature. Higher market participation reduces price volatility by increasing market liquidity, which allows trades to be absorbed with less price impact (Pagano, 1989; Allen and Gale, 1994). See Knaut and Paschmann (2019) for empirical evidence from energy spot markets.

4.5 Futures

Assume that a fraction θ_H of connected agents are hedgers, while $1-\theta_H$ are speculators. Additionally, suppose a market-making company facilitates futures trading. At the end of each period, hedgers with productivity z contract to sell all their assets at the beginning of the next period to the market maker for a price of $m_0(z)$. Speculators hold shares in the market maker proportional to their monetary holdings. As a result, hedgers' monetary holdings are unchanged after the shock, making them formally identical to unconnected agents. Meanwhile, speculators receive a proportional shock of $\tilde{\tau} = (1 + M^H_{c0}/M^S_{c0})\tau > \tau$, where M^H_{c0} and M^S_{c0} denote the average monetary holdings of hedgers and speculators, respectively.

Due to perfect foresight, speculators' higher risk does not command a premium in the stationary equilibrium. Thus, introducing a futures market is isomorphic to lowering η, as only a fraction $\tilde{\eta}_c = \eta(1-\theta_H) < \eta$ receives monetary transfers from the government. This implies that futures trading concentrates monetary policy risk on fewer agents, slowing the transition and making prices more sluggish. The impact on output volatility follows an inverse U-shape: it may rise for high $\tilde{\eta}_c$ but can stabilize if speculators are few. Welfare effects remain ambiguous, depending on the productivity distribution between hedgers and speculators.

4.6 Varying Access To Financial Markets and the Aggregate Shock

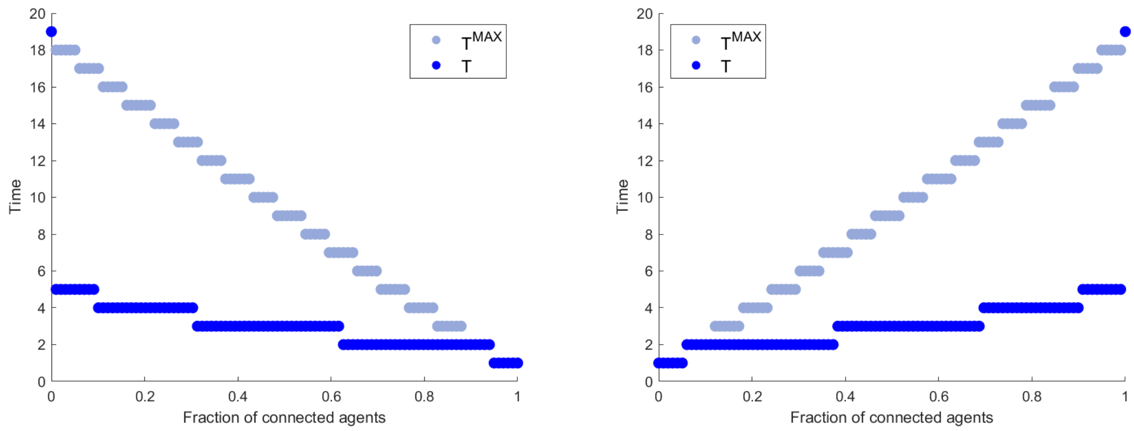
The analysis above keeps the shock to the monetary base fixed while varying the fraction of connected agents, η. However, one could argue that, for the same shock to the policy rate, the idiosyncratic shock, τ, would remain fixed[17] while the aggregate shock would vary[18]. I will now analyze this case. For $\eta \in \{0.1, 0.27, 0.5, 0.75\}$, we have $\tau^A \in \{0.02, 0.054, 0.1, 0.15\}$. Figure 12d, in Appendix C, shows that a larger shock leads to a higher price level. Output declines the least at very high and very low η, but the transition is shorter as the economy moves closer to helicopter drops. Besides, Table 8, in Appendix C, shows that welfare losses follow an inverse U-shape in η. Hence, when access to financial markets is widespread, the same interest cut is more effective in affecting inflation, and monetary policy lags would be smaller. However, shocks can be highly distortionary for intermediate levels of financial market access.

[17] This is clearer in the context of a government bond market (not modeled here), where prices would rise uniformly across scenarios for a given policy rate cut.

[18] If, as argued before, a larger shock is needed to move prices in a thick asset market, it follows that the money supply shock should increase with η.

4.7 Varying The Extensive And Intensive Margins Of Inequality

Now, I conduct a similar exercise by varying η and maintaining the aggregate shock, $\tau^A = 0.135$. This time, I set the individual shock at $\tau = 0.2$ and vary the relative average productivity of connected and unconnected.[19] I consider $M_{c0}/M_0 \in \{7.56, 2.8, 1.512, 1.008\}$ for, respectively, $\eta \in \{0.1, 0.27, 0.5, 0.75\}$. This exercise trades off two dimensions of inequality: the extensive margin (the number of connected agents) and the intensive margin (how much richer they are relative to the unconnected). As can be seen in Figure 13, aggregate variables and individual choices remain unchanged across cases, except for the paths of TFP and Gini coefficients. Naturally, for higher values of η, the productivity advantage of connected agents is lower by construction, reducing fluctuations in allocative efficiency and inequality.



(a) Positive shock, $\tau = 0.2$ (b) Negative shock, $\tau = -0.1667$

Figure 5: Length of the transition path for different fractions of connected

4.8 Financial Access And The Length of the Transition Path

I now allow the fraction of connected to vary more freely over the interval $\eta \in [0, 1]$.[20] Individual shocks remain fixed — either positive or negative, $\tau \in \{0.2, -0.1667\}$ — as do relative initial average monetary holdings.[21] As a consequence, η governs the size of the aggregate shock. Figure 5 plots both T^{MAX}, computed as in (27), and the

[19]The aggregate shock is now higher than before because I assume $M_{c0}/M_0 = 2.8$ in the economy with $\eta = 0.27$ to ensure that connected agents are, on average, more productive than the unconnected across all scenarios.

[20]When $\eta = 0$ ($\eta = 1$), I consider a discrete number of connected (unconnected) agents, with zero mass, meaning no aggregate shock (nearly helicopter drops).

[21]I do not recalibrate the model, as the normalization of the initial aggregate price is irrelevant here. For simplicity, I set $M_{c0}/M_0 = 1$.

length of the transition path for both shock signs. The graphs indicate that general equilibrium effects explain most of the length of the transition path. Specifically, when connected (unconnected) agents have zero mass, $T = T^{MAX}$ for a positive (negative) monetary shock. Moreover, the gap between T and T^{MAX} shrinks when high-cash agents are more numerous. Intuitively, for a given individual shock, a bigger aggregate shock raises the expected market revenues of high-cash agents, reducing their incentives to smooth consumption over an extended period.

4.9 Collateral constraint

So far, I have examined the post-shock dynamics in scenarios in which borrowing is either not possible, unconstrained, or subject to a rationing constraint due to a nominal interest rate peg. My focus has thus been on the extensive margin of credit market access. I now soften the financial frictions by examining them along the intensive margin. This not only provides a middle ground between the bondless and full enforcement economies but also aligns more closely with much of the literature on financial frictions. I define a borrowing constraints as follows:

$$b_{i,t+1}(z) \geq -l_t(z, m_{it}(z)) = -\kappa R_{it}(z), \tag{37}$$

with $\kappa \in (0,1)$. Naturally, if it does not bind for any agent, the economy behaves as it would under full enforcement. The proposition below characterizes, for log utility, the post-shock convergence properties of such an economy:

Proposition 5. *For log utility, there is a $T < \infty$ at which the economy with a borrowing constraint achieves a stationary equilibrium, which can be either:*

- *Non-fundamental if $b_{l,T+1}(z) = b^*_{l,T+1}(z) \geq -\kappa R^*_{lT}(z)$ for $z \in [\underline{z}, \overline{z}]$;*
- *Fundamental if $b_{l,T+1}(z) = 0$ and $b^*_{l,T+1}(z) < -\kappa R^*_{lT}(z)$ and $\frac{\beta}{q_T} m_{hT}(z) \leq \theta_0(z) M_{T+1}$ for $z \in [\underline{z}, \overline{z}]$,*

where the $*$ superscript denotes the variable's unconstrained level. Along the transition, $1 \geq q_t > \beta$. If the economy exits the zero lower bound at t, it reaches the stationary equilibrium at $t+1$.

The proposition states that the economy attains a stationary equilibrium in finite time. Whether this equilibrium is fundamental depends on conditions at the last period of the transition, T. Intuitively, as low-cash agents replenish their money over time and reduce their bond supply, the borrowing constraint may cease to bind. This occurs if the optimal bond sales, $|b^*_{l,T+1}(z)|$, fall below the borrowing limit. If, however, before low-cash agents reach that point, high-cash agents decide to fully deplete their money, the economy goes to the fundamental stationary equilibrium.

The stationary equilibrium depends on the severity of financial frictions. In either case, inequality in monetary holdings declines along the transition. The economy may hit the zero lower bound if there is excess demand for bonds at any positive interest rate. Although entrepreneurs still have a preference for bonds by assumption, they are compelled to maintain positive cash savings, since bond sales fall short of their demand. Still, for any $\kappa > 0.0995$, the collateral constraint never binds, implying that distortions from collateral requirements are rare in the model. Thus, the intensive margin of credit access likely has little impact in real economic settings.

5 Concluding remarks

This paper has shown that the distributional effects of monetary shocks can delay the economy's return to long-run equilibrium. When a well-functioning credit market is absent, the transition features sluggish price adjustments and depressed output due to idle cash balances. A well-functioning bond market mitigates consumption inequality but introduces hysteresis in monetary holdings, preventing a full return to fundamental equilibrium unless collateral constraints bind or interest rates deviate from their equilibrium level.

My contributions lie in three main areas. First, I provide a tractable general equilibrium model linking portfolio and general equilibrium channels, allowing for endogenous restoration of original allocations and the study of convergence properties. Second, I demonstrate that even limited credit access significantly alters monetary policy transmission by enabling near-complete risk sharing. Third, I offer new insights into price stickiness, which can be partially driven by (i) beneficiaries of monetary shocks lowering marginal costs and (ii) others setting lower prices to restore wealth.

Four key policy implications emerge. To begin, reducing financial frictions that hinder access to credit substantially enhances market completeness, while easing collateral constraints is less effective. Besides, broader access to financial markets dilutes post-shock distortions under money-supply targeting. However, under interest rate targeting, larger liquidity shocks may be required when participation is high, leading to an inverse U-shaped effect on distortions. Furthermore, credit markets alleviate short-term distortions but can create persistent non-fundamental wealth disparities, highlighting the need for complementary inequality-reducing policies. Lastly, given that the welfare losses from distortions are small in the long run, distributional effects are largely irrelevant to monetary policy in developed economies with well-functioning credit markets.

Several extensions merit further exploration. First, a more microfounded approach to financial segmentation could refine the link between interest rate shocks, asset prices, and credit market dynamics. Besides, persistent inflation influences the nominal interest rate through the liquidity effect, impacting the persistence of idiosyncratic shocks under full enforcement of bond contracts. Thus, exploring a non-zero money supply growth rate could provide further insights. Finally, introducing information and pricing frictions could provide further insights into monetary transmission under incomplete markets.

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Appendix

A Proofs of Propositions

A.1 Proposition 1

Existence and uniqueness of the fundamental stationary equilibrium:

Notice that, for any arbitrary $z \in [\underline{z}, \overline{z}]$ and $i \in \{c, u\}$, by imposing the fundamental stationary equilibrium conditions onto (3), we obtain:

$$P_t C_{it}(z) = m_{it}(z) + (1 - q_t) b_{i,t+1}(z).$$

Imposing the cash-in-advance constraint, (2), gives:

$$m_{it}(z) - q_t b_{i,t+1}(z) = m_{it}(z) + (1 - q_t) b_{i,t+1}(z),$$

which can only be satisfied for $b_{i,t+1}(z) = 0$. Now, by aggregating (13), and imposing $P_t = P_{t+1}$ and $C_{i,t+1}(z) = C_{it}(z)$ for every $z \in [\underline{z}, \overline{z}]$ and $i \in \{c, u\}$, we find that:

$$\left[\sum_{i \in \{c,u\}} \eta_i \int_{\underline{z}}^{\overline{z}} z^{\epsilon-1} \frac{u'\left(C_{it}(z)\right)^{\epsilon-1}}{h_{it}(z)^{\zeta(\epsilon-1)}} dF_i(z) \right]^{\frac{1}{\epsilon-1}} = \left(\frac{\epsilon}{\epsilon - 1} \right) \frac{\gamma}{\beta} \tag{38}$$

By exploiting the continuity of the $u'(\cdot)$, I define z^* as being the $z \in [\underline{z}, \overline{z}]$ that satisfies:

$$\frac{(z^*)^{\epsilon-1} u'\left(C_{it}(z^*)\right)^{\epsilon-1}}{h_{it}(z^*)^{\zeta(\epsilon-1)}} = \sum_{i \in \{c,u\}} \eta_i \int_{\underline{z}}^{\overline{z}} z^{\epsilon-1} \frac{u'\left(C_{it}(z)\right)^{\epsilon-1}}{h_{it}(z)^{\zeta(\epsilon-1)}} dF_i(z) \tag{39}$$

Now, notice that:

$$\theta_{it}(z) = \left[\frac{z u'\left(C_{i,t+1}(z)\right) / h_{it}(z)^{\zeta}}{z^* u'\left(C_{i,t+1}(z^*)\right) / h_{it}(z^*)^{\zeta}} \right]^{\epsilon-1},$$

which follows from (38) and (39). This means that $\theta_{it}(z^*) = 1$. Since, in the fundamental stationary equilibrium, there is no borrowing and $m_{it}(z) = R_{it}(z)$ for every $z \in [\underline{z}, \overline{z}]$ and $i \in \{c, u\}$, we must have $C_{i,t+1}(z^*) = C_{it}(z^*) = \theta_{it}(z^*) M_t / P_t = M_t / P_t$. This means that:

$$\left(\frac{\epsilon}{\epsilon - 1} \right) \frac{\gamma}{\beta} = z^* \frac{u'\left(M_t / P_t\right)}{h_{it}(z^*)^{\zeta}} \tag{40}$$

Moreover, with a bit of algebra, we can show that:

$$h_{it}(z) = \frac{1}{z} \left(\frac{p_{it}(z)}{P_t} \right)^{-\epsilon} Y_t = \left[\left(\frac{\epsilon - 1}{\epsilon} \right) \frac{1}{\gamma} \beta \frac{P_t}{P_{t+1}} u'(C_{i,t+1}(z)) \right]^{\frac{\epsilon}{1+\zeta\epsilon}} z^{\frac{\epsilon-1}{1+\zeta\epsilon}} Y_t^{\frac{1}{1+\zeta\epsilon}}, \tag{41}$$

By using (41) and the fact that, in the fundamental stationary equilibrium, $Y_t = M_t/P_t$ and $P_{t+1} = P_t$, one can show that:

$$u'\left(\frac{M_t}{P_t}\right)\left[\frac{M_t}{P_t}\right]^{-\zeta} = \left(\frac{\epsilon}{\epsilon - 1}\right)\frac{\gamma}{\beta(z^*)^{1+\zeta}}, \tag{42}$$

which means that M_t/P_t is defined uniquely, as the left-hand side is strictly decreasing on it. Therefore, P_t is proportional to M_t, and aggregate output $Y_t = M_t/P_t$ is defined by parameters and z^* alone, being, therefore, independent of P_t and M_t. Finally, $\theta_{it}(z)$ is also well and uniquely defined for every $z \in [\underline{z}, \overline{z}]$ and is independent of connectedness status. This means that $m_{it}(z)$, $C_{it}(z)$, $p_{it}(z)$, $y_{it}(z)$ and $R_{it}(z)$ are also well- and uniquely-defined. Now, I look into non-fundamental stationary equilibria.

Finding a stationary equilibrium with $q_t = \beta$:

Let us define "high-cash" agents as being agents for whom $b_{h,t+1}(z) > 0$ and "low-cash" as having $b_{l,t+1}(z) \leq 0$. We can then build $F_h(\cdot)$ and $F_l(\cdot)$ to be the corresponding cumulative distribution functions. Finally, let η_h and $\eta_l = 1 - \eta_h$ be the corresponding fractions of the population that falls into either category.

The structure of the proof is as follows: I will begin by assuming that $l_t(z, m_{lt}(z)) = \infty$. This allows some low-cash agents to end up with an outstanding debt after the bonds market closes at the beginning of the period. Then, I will show that the equilibrium implemented by the unconstrained economy is feasible in the constrained economy. First, notice that the Euler equation in the unconstrained case is given by:

$$\frac{u'(C_{it}(z))}{u'(C_{i,t+1}(z))} = \frac{\beta}{q_t}\frac{P_t}{P_{t+1}} \tag{43}$$

Since the right-hand side is common to all agents, either $C_{it}(z)/C_{i,t+1}(z)$ is decreasing, constant, or increasing over time for everyone. If $q_t = 1$, $s_{it}(z) = 0$ for all agents by assumption, and, if $q_t < 1$, positive cash savings, $s_{it}(z) > 0$, cannot be optimal, meaning that high-cash agents' savings is channeled to low-cash. Then, $M_t^C = M_t$ for $t \in \{1, 2, ...\}$. Now, I will show that there is a stationary equilibrium with $q_t = \beta$. To begin, if we set $q_t = \beta$, (43) implies that $\frac{u'(C_{i1}(z))}{u'(C_{i2}(z))} = \frac{P_1}{P_2}$. Moreover:

$$P_t \sum_{i \in \{h,l\}} \eta_i \int_{\underline{z}}^{\overline{z}} C_{it}(z) dF_i(z) = M_t = P_{t+1} \sum_{i \in \{h,l\}} \eta_i \int_{\underline{z}}^{\overline{z}} C_{i,t+1}(z) dF_i(z), \tag{44}$$

Naturally, if $C_{it}(z) = C_{i,t+1}(z)$ for every $z \in [\underline{z}, \overline{z}]$ and $i \in \{h, l\}$, the equation above implies that $P_t = P_{t+1}$, meaning that the first-order condition (43) is satisfied for every agent. Lastly, integrating (2) implies $\sum_{i \in \{h,l\}} \eta_i \int_{\underline{z}}^{\overline{z}} b_{i,t+1}(z) dF_i(z) = 0$, meaning that the bonds market is equilibrium. I will prove below that this requires $m_{it}(z) = m_{i,t+1}(z)$ for all agents and for any arbitrary $t = 1, 2, ...$, but, for now, I will take this result as a given for simplicity. It must be the case that this equilibrium lasts forever, proving the existence of this stationary equilibrium with borrowing. Let us denote the demand/supply of bonds by each agent under this equilibrium with a star, that is, $b^*_{it}(z)$

Proving that $q_t = \beta$ for every period t:

Now assume, by contradiction, that $1 \geq q_t > \beta$. The argument above proves that an equilibrium with $q_t = \beta$ always exists, which means that $\sum_{i\in\{h,l\}} \eta_i \int_{\underline{z}}^{\overline{z}} b^*_{i,t+1}(z)dF_i(z) = 0$. I now prove that there cannot be an equilibrium with $q_t \neq \beta$. There are three cases. First, consider $q_t \frac{P_{t+1}}{P_t} > \beta$, meaning that the real interest rate is below the one in the stationary equilibrium described above. By (43), $u'(C_{i,t+1}(z)) > u'(C_{it}(z))$ for all $z \in [\underline{z}, \overline{z}]$ and $i \in \{h,l\}$. This implies that $P_{t+1} > P_t$ to ensure that the amount of money in circulation equates to the whole monetary base, $M^C_{t+1} = M^C_t$. Since there is inflation and the real interest rate is lower than the one in the equilibrium above, bond demand by the high-cash agents must be lower than in the equilibrium above, *i.e.*, $b_{h,t+1}(z) < b^*_{h,t+1}(z)$, and low-cash agents' supply of bonds must be higher, *i.e.*, $-b_{l,t+1}(z) > -b^*_{l,t+1}(z)$. This implies:

$$0 = \sum_{i\in\{h,l\}} \eta_i \int_{\underline{z}}^{\overline{z}} b_{i,t+1}(z)dF_i(z) < \sum_{i\in\{h,l\}} \eta_i \int_{\underline{z}}^{\overline{z}} b^*_{i,t+1}(z)dF_i(z) = 0, \tag{45}$$

which means that the bonds market is not in equilibrium, a contradiction.

Second, the case of $q_t \frac{P_{t+1}}{P_t} < \beta$ is analogous, since it implies $u'(C_{i,t+1}(z)) < u'(C_{it}(z))$ for $z \in [\underline{z}, \overline{z}]$ and $i \in \{h,l\}$, by (43), and $P_{t+1} < P_t$, by $M^C_{t+1} = M^C_t$. Hence, there is deflation and a real interest rate above the one in the equilibrium described above. As a result, $b_{h,t+1}(z) > b^*_{h,t+1}(z)$ and $-b_{l,t+1}(z) < -b^*_{l,t+1}(z)$. This, in turn, implies that $\sum_{i\in\{h,l\}} \eta_i \int_{\underline{z}}^{\overline{z}} b_{i,t+1}(z)dF_i(z) > 0$, and, hence, the economy is not in equilibrium. Third, we consider the case where $q_t \frac{P_{t+1}}{P_t} = \beta$. This implies $u'(C_{i,t+1}(z)) = u'(C_{it}(z))$ for $z \in [\underline{z}, \overline{z}]$ and $i \in \{h,l\}$ and, hence, $P_{t+1} = P_t$. However, since $q_t > \beta$, $q_t \frac{P_{t+1}}{P_t} = \beta$ implies $P_t > P_{t+1}$, a contradiction. This rules out the possibility of an equilibrium with $1 \geq q_t > \beta$. A similar argument can be made to rule out equilibria with $q_t < \beta$.

Equilibrium uniqueness:

Now, I will show that, under $q_t = \beta$, there can be no equilibrium that is not stationary. This will prove the uniqueness of the equilibrium described above under full enforcement. By integrating the individual price choices, we obtain:

$$P_t = P_{t+1} \left(\frac{\epsilon}{\epsilon - 1}\right) \frac{\gamma}{\beta} \left[\sum_{i\in\{h,l\}} \eta_i \int_{\underline{z}}^{\overline{z}} \frac{z^{\epsilon-1} u'(C_{i,t+1}(z))^{\epsilon-1}}{h_{it}(z)^{\zeta(\epsilon-1)}} dF_i(z) \right]^{\frac{1}{1-\epsilon}} \tag{46}$$

Plugging (41) into the equation above yields:

$$P_t = P_{t+1}\left(\frac{\epsilon}{\epsilon-1}\right)\frac{\gamma}{\beta}Y_t^{\zeta}\left[\sum_{i\in\{h,l\}}\eta_i\int_{\underline{z}}^{\overline{z}} z^{\frac{(\epsilon-1)(1+\zeta)}{1+\zeta\epsilon}}u'(C_{i,t+1}(z))^{\frac{\epsilon-1}{1+\zeta\epsilon}}dF_i(z)\right]^{\frac{1+\zeta\epsilon}{1-\epsilon}} \tag{47}$$

Now, notice that, at any period, and since $q_t = \beta$, (43) can be written with a weak inequality, that is $u'(C_{i,t+1}(z)) \leq \frac{P_{t+1}}{P_t}u'(C_{it}(z))$, which should also be valid even if the economy ends up in a fundamental stationary equilibrium in the next period. By plugging this version of the first-order condition into the equation above, we obtain:

$$\left[\sum_{i\in\{h,l\}}\eta_i\int_{\underline{z}}^{\overline{z}} z^{\frac{(\epsilon-1)(1+\zeta)}{1+\zeta\epsilon}}u'(C_{it}(z))^{\frac{\epsilon-1}{1+\zeta\epsilon}}dF_i(z)\right]^{\frac{1+\zeta\epsilon}{\epsilon-1}} \geq \left(\frac{\epsilon}{\epsilon-1}\right)\frac{\gamma}{q_t}Y_t^{\zeta} = \left(\frac{\epsilon}{\epsilon-1}\right)\frac{\gamma}{\beta}Y_t^{\zeta}$$

which is valid for any t. This allows us to re-write (47) as:

$$P_t \leq P_{t+1}\left(\frac{Y_t}{Y_{t+1}}\right)^{\zeta}, \tag{48}$$

Since $P_t = M_t/Y_t$ and $M_{t+1} = M_t$, this condition implies $Y_t \geq Y_{t+1}$. Notice that this immediately rules out the possibility of deflation. Thus, we must either have inflation or constant prices. Assume, by contradiction, that $P_{t+1} > P_t$ for a given period $t \in \{1, 2, ...\}$. Then, by using the definition (16) and (46), we can see that:

$$\overline{U}_{t+1}^{GAP} = \frac{P_{t+1}}{P_t} > 1, \tag{49}$$

I am going to show that this implies that $P_{t+2} > P_{t+1}$. To see this, notice that, since $P_{t+2} \geq P_{t+1}$ by the condition above, and given the first-order condition (43) and $q_t = \beta$, then $C_{i,t+2}(z) \leq C_{i,t+1}(z)$ for any $z \in [\underline{z}, \overline{z}]$ and $i \in \{h, l\}$. Imposing (41) yields:

$$\begin{aligned}\frac{z^{\epsilon-1}u'(C_{i,t+2}(z))^{\epsilon-1}}{h_{i,t+1}(z)^{\zeta(\epsilon-1)}} &> \left[\left(\frac{\epsilon}{\epsilon-1}\right)\frac{\gamma}{\beta}\frac{P_{t+2}}{P_{t+1}}\right]^{\frac{\zeta\epsilon(\epsilon-1)}{1+\zeta\epsilon}} Y_t^{\frac{\zeta(1-\epsilon)}{1+\zeta\epsilon}} z^{\frac{(\epsilon-1)(1+\zeta)}{1+\zeta\epsilon}}u'(C_{i,t+1}(z))^{\frac{\epsilon-1}{1+\zeta\epsilon}} \\ &= \frac{z^{\epsilon-1}u'(C_{i,t+1}(z))^{\epsilon-1}}{h_{it}(z)^{\zeta(\epsilon-1)}}\left(\frac{P_{t+2}}{P_{t+1}}\frac{P_t}{P_{t+1}}\right)^{\frac{\zeta\epsilon(\epsilon-1)}{1+\zeta\epsilon}}\end{aligned}$$

since $Y_{t+1} < Y_t$ due to our contradiction assumption coupled with $M_{t+1}^C = M_t^C$. By (49), we obtain $\frac{P_{t+2}}{P_{t+1}} > \frac{P_{t+1}}{P_t}$, which shows that prices cannot be constant. By an induction argument, we can see that the inflation rate is bounded below by a positive constant, since $1 + \pi_{t+s} = \overline{U}_{t+s}^{GAP} \geq \overline{U}_{t+1}^{GAP} > 1$ for $s \in \{2, 3, ...\}$, meaning that the price level diverges. This, implies, however, by equation (44), that:

$$\lim_{t\to\infty}\sum_{i\in\{h,l\}}\eta_i\int_{\underline{z}}^{\overline{z}} C_{it}(z)dF_i(z) = 0$$

By the fact that there is borrowing in any equilibrium along this path, there is a non-negligible mass of high-cash agents that are strictly richer than their low-cash counterparts, that is, $C_{ht}(z) > C_{lt}(z) \geq 0$, meaning that aggregate consumption must be bounded away from zero. Thus, the only possible equilibrium is the one in which $P_{t+1} = P_t$ and $C_{i,t+1}(z) = C_{it}(z)$ for every $z \in [\underline{z}, \overline{z}]$, $i \in \{h, l\}$ and $t \in \mathbb{T}^S$. Evidently, $P_t C_{ht}(z) > P_t C_{lt}(z)$, $p_{ht}(z) > p_{lt}(z)$ and, thus, $R_{lt}(z) > R_{ht}(z)$ for all $t \in \mathbb{T}^S$.

Constancy of borrowing/lending decisions:

By imposing (2) into the budget constraint, (3), we see that $m^-_{i,t+1}(z) = R_{it}(z)$. Now, notice that, since $u'(C_{i,t+1})/P_{t+1} = u'(C_{i,t+2})/P_{t+2}$ for $z \in [\underline{z}, \overline{z}]$ and $i \in \{h, l\}$, by (13), $p_{it}(z) = p_{i,t+1}(z)$ and, thus, $R_{it}(z) = R_{i,t+1}(z)$. This means that $m^-_{it}(z) = m^-_{i,t+1}(z)$ for $t = 2, 3, ...$ Therefore, by the budget constraint, (3), we have:

$$P_t C_{it}(z) = R_{it}(z) + b_{it}(z) - \beta b_{i,t+1}(z) \tag{50}$$

Together with (50) and the fact that $P_t C_{it}(z) = P_{t+1} C_{i,t+1}(z)$, this implies that:

$$b_{it}(z) - \beta b_{i,t+1}(z) = b_{i,t+1}(z) - \beta b_{i,t+2}(z) \tag{51}$$

for every $t \in \mathbb{T}^S$. Now, let us define α_s such that $b_{i,t+s}(z) = \alpha_s b_{i,t+s-1}(z)$, for $s = 1, 2, ...$ Thus, for an arbitrary $r = 1, 2, ...$, we can apply this definition iteratively to obtain:

$$b_{i,t+r}(z) = \left(\prod_{s=1}^{r} \alpha_s \right) b_{it}(z) \tag{52}$$

I will now prove that $\alpha_s = 1$ for $s = 1, 2, ...$ I will concentrate on α_1 without any loss of generality. Assume, by contradiction, that $\alpha_1 < 1$. Then, by (51), we have:

$$(1 - \alpha_1 \beta) = (1 - \alpha_2 \beta)\alpha_1 < (1 - \alpha_2 \beta) \qquad \therefore \qquad \alpha_2 < \alpha_1$$

Obviously, the same reasoning applies to show that $\alpha_s < \alpha_1$. Therefore, $\alpha_s < 1$ for all $s \geq 1$. However, this means that $\lim_{r\to\infty} b_{i,t+r}(z) = 0$ and, thus:

$$\lim_{r\to\infty} P_{t+r} C_{i,t+r}(z) = \lim_{r\to\infty} R_{i,t+r}(z)$$

However, since $R_{l,t+r}(z) > R_{h,t+r}(z)$, this means that $P_{t+r} C_{l,t+r}(z) > P_t C_{h,t+r}(z)$ at the limit: a contradiction, since $P_{t+r} C_{h,t+r}(z)$ is constant for all r and larger than $P_{t+r} C_{l,t+r}(z)$.

Now, I proceed to the second case: assume, by contradiction, that $\alpha_1 > 1$. Similarly, this implies that $\alpha_s > \alpha_1 > 1$ for any $s > 1$. Now, this means that:

$$\lim_{r\to\infty} b_{c,t+r} = \lim_{r\to\infty} \left(\prod_{s=1}^{r} \alpha_s \right) b_{ct} > \lim_{r\to\infty} \alpha_1^r b_{ct} = \infty$$

which is not possible, since $M_t < \infty$. Therefore, I conclude that $\alpha_s = 1$ for $s = 1, 2, ...$ and, thus, $b_{it}(z) = b_{i,t+1}(z)$ for $z \in [\underline{z}, \overline{z}]$, $i \in \{h, l\}$ and $t \in \mathbb{T}^S$.

Proving that the unconstrained optimum is feasible:

Using agents' cash-in-advance constraint, (2), at t, this result implies that:

$$b_{it}(z) = \frac{m_{it}(z) - P_t C_{it}(z)}{\beta}$$

for $z \in [\underline{z}, \overline{z}]$ and $i \in \{h, l\}$. One can easily see that $m_{it}(z) = m_{i,t+1}(z)$ for $t \in \mathbb{T}^S$. Since $m_{i1}(z) > 0$ for every $z \in [\underline{z}, \overline{z}]$ and $i \in \{h, l\}$, this implies that no agent enters the bonds market with outstanding debt, meaning that $b_{l,t+1}(z) < m^{-}_{l,t+1}(z) = R_{lt}(z)$ for every $z \in [\underline{z}, \overline{z}]$, proving that the borrowing constraint does not bind.

Fundamental Stationary Equilibrium Uniqueness:

Finally, I can prove that, if money is distributed such as in the fundamental stationary equilibrium, it is the unique equilibrium in this economy. I denote the fundamental stationary equilibrium with the subscript 0 and, moreover, drop the i, since choices do not depend on the entrepreneur's type. The only circumstance in which the economy would not be in the fundamental stationary equilibrium is if $P_t C_{it}(z) < m_0(z)$ for a non-negligible mass of agents. Assume this is the case by means of contradiction. This signifies that $b_{i,t+1}(z) \neq 0$ for a positive mass of agents.

As proven above, under $l_t(z, m_t(z)) = m_{t+1}(z)$, $q_t = \beta$, meaning that $\frac{u'(C_{i,t+1}(z))}{P_{t+1}} = \frac{C_{it}(z)}{P_t}$ for every $i \in \{h, l\}$ and $z \in [\underline{z}, \overline{z}]$ by (43). Consider the case of a (possibly counterfactual) non-lender counterpart — that is, an agent who chooses $C_{it}(z) = \frac{m_0(z)}{P_t}$ — to the agents who buy bonds, with the same productivity. By plugging (41) into (13), we obtain:

$$\frac{\theta_{non-lender,t}(z)}{\theta_{lender,t}(z)} = \left[\frac{u'\left(C_{non-lender,t+1}(z)\right)}{u'\left(C_{lender,t+1}(z)\right)}\right]^{\frac{\epsilon-1}{1+\zeta\epsilon}} = \left[\frac{u'\left(C_{non-lender,t}(z)\right)}{u'\left(C_{lender,t}(z)\right)}\right]^{\frac{\epsilon-1}{1+\zeta\epsilon}} < 1,$$

since $P_t C_{lender,t}(z) < m_0(z) = P_t C_{non-lender,t}(z)$. Given that $M_t^C = M_t$, this means that $R_{lender,t}(z) > R_{non-lender}(z)$. However, since $b_{lender,t+1}(z) > 0 = b_{non-lender,t+1}(z)$, plugging (2) into (3) yields:

$$m_{lender,t+1}(z) = R_{lender,t+1}(z) + b_{lender,t+1}(z) > R_{non-lender,t+1}(z) = m_{non-lender,t+1}(z),$$

which means that the lender should consume more than the non-lender, that is, they should borrow money, a contradiction. I conclude that $m_t(z) = R_0(z)$ implies that the economy is in the fundamental stationary equilibrium. □

A.1.1 Corollary 1.1

By assumption, the economy starts off at the fundamental stationary equilibrium. The monetary base grows, that is, $M_1 = (1+\tau^A)M_0$. If revenues are undistorted relative to the fundamental equilibrium, $m_1(z) = (1+\tau_H)m_0(z) = \theta_0(z)(1+\tau^A)M_0 = R_1(z)$, since $\tau_H = \tau^A$. However, this implies that the new monetary holdings are compatible with a new fundamental stationary equilibrium. Given Proposition 1, this is the unique equilibrium that can arise in this economy. Now, notice that, even if borrowing was possible, in the fundamental equilibrium, $b_{t+1}(z) = 0$ for all $z \in [\underline{z}, \overline{z}]$. Since buying bonds dominates saving in idle cash for $q_t = \beta$, this implies that $s_t(z) = 0$ for all $z \in [\underline{z}, \overline{z}]$ in the economy where $l_t(z, m_t(z)) = 0$. This proves that the economy must be in the fundamental stationary equilibrium after the helicopter drops shock. $\square$

A.1.2 Corollary 1.2

Since $P_{t+1}C_{i,t+1}(z) = R_{it}(z)$, by (13), with some algebra, we must have:

$$p_{it}(z) = \left(\frac{\epsilon}{\epsilon-1}\right)\frac{\gamma}{\beta}\left[\frac{D(p_{it}(z), P_t, Y_t)}{z}\right]^{1+\zeta} \quad p_{it}(z) = \left[\left(\frac{\epsilon}{\epsilon-1}\right)\frac{\gamma}{\beta}\frac{Y_t^{1+\zeta}}{z^{1+\zeta}}\right]^{\frac{1}{\epsilon(1+\zeta)}} P_t \tag{53}$$

for $t \in \mathbb{T}^S$. Aggregating prices according to (12) and using $P_t = \frac{M_t}{Y_t}$ yields:

$$Y_t = \left[\left(\frac{\epsilon-1}{\epsilon}\right)\frac{\beta}{\gamma}\right]^{\frac{1}{1+\zeta}} \mathcal{Z} \quad \text{and} \quad P_t = \left[\left(\frac{\epsilon}{\epsilon-1}\right)\frac{\gamma}{\beta}\right]^{\frac{1}{1+\zeta}} \frac{M_t}{\mathcal{Z}} \tag{54}$$

for $t \in \mathbb{T}^S$. Using (53), and plugging (54) into it gives:

$$\theta_{it}(z) = \left(\frac{p_{it}(z)}{P_t}\right)^{1-\epsilon} = \frac{z^{\frac{\epsilon-1}{\epsilon}}}{\mathcal{Z}^{\frac{\epsilon-1}{\epsilon}}}$$

Plugging this into $m_{it}(z) = \theta_t(z)M_t$, $C_{it}(z) = \frac{m_{it}(z)}{P_t}$ and $p_{it}(z) = \theta_{it}(z)^{\frac{1}{1-\epsilon}} P_t$ yields the remaining equations. $\square$

A.1.3 Corollary 1.3

At $t = 1$, the cash-in-advance and the budget constraints, (2) and (3), are:

$$P_1 C_{i1}(z) = m_{it}(z) - \beta b_{i2}(z)$$
$$P_2 C_{i2}(z) = R_{i1}(z) + (1-\beta) b_{i2}(z)$$

Since $P_t C_{it}(z) = P_s C_{is}(z)$ for $i \in \{h, l\}$ and $t, s = 1, 2, ...$, re-arranging the terms yields:

$$P_t C_{ct}(z) = (1-\beta)(1+\tau)m_0(z) + \beta R_{ct}(z)$$
$$P_t C_{ut}(z) = (1-\beta)m_0(z) + \beta R_{ut}(z)$$

Subtracting the latter expression from the former yields:

$$P_t C_{ct}(z) - P_t C_{ut}(z) = (1-\beta)\tau m_0(z) - \beta(R_{ut}(z) - R_{ct}(z))$$

Since we know that $C_{ct}(z) > (<) C_{ut}(z)$, then $P_t C_{ct}(z) - P_t C_{ut}(z) > (<) 0$ for $\tau > (<) 0$. As a result, we obtain:

$$|R_{ut}(z) - R_{ct}(z)| < \left| \left(\frac{1-\beta}{\beta} \right) \tau m_0(z) \right|$$

Equations (34) and (35) follow immediately from the facts that $R_{ut}(z) - R_{ct}(z) > (<) 0$ and $C_{ct}(z) > (<) C_{ut}(z)$ for $\tau > (<) 0$. □

A.2 Proposition 2

Assume that, if no high-cash agent chooses to partially deplete, then $T = 1$ and, hence, the result follows trivially. I will, then, concentrate on the case where a positive mass of high-cash agents partially depletes. The case where a zero mass of high-cash agents partially depletes also follows as a combination of both cases.

Low-cash agents are more likely to fully deplete their cash at $t = 1$:
First, I need to prove that, for any $z \in [\underline{z}, \overline{z}]$, the low-cash agent is more likely to fully deplete at $t = 1$. Assume that the high-cash agent saves nothing at $t = 1$. Then $C_{h1}(z) = \frac{m_{h1}(z)}{P_1} > \frac{m_{l1}(z)}{P_1} \geq C_{l1}(z)$. By the first order condition of the — fully-depleting — high-cash agent:

$$\frac{u'(C_{l1}(z))}{P_1} > \frac{u'(C_{h1}(z))}{P_1} \geq \beta \frac{u'(R_{h1}(z)/P_2)}{P_2},$$

Since $R_{l1}(z) = R_{h1}(z)$ in the case where both fully deplete at $t = 1$, the full depletion condition is satisfied for the low-cash agent as well. So, the low-cash agent always fully depletes when the high-cash one with the same productivity does, proving our result. Thus, for any $z \in [\underline{z}, \overline{z}]$, there are three possibilities: 1) both kinds of agent fully deplete; 2) both partially deplete; or 3) only the high-cash type partially depletes.

Revenues are not lower for low-cash agents than for high-cash ones:
I will show that $\theta_{l1}(z) \geq \theta_{h1}(z)$. We analyze each of the cases in turn:

1) Everyone fully depletes for $z \in [\underline{z}, \overline{z}]$: Naturally, $\theta_{l1}(z) = \theta_{h1}(z)$.

2) Everyone partially depletes for $z \in [\underline{z}, \overline{z}]$: Now, we have, for $i \in \{h, l\}$:

$$\theta_{i1}(z) = \left[\left(\frac{\epsilon - 1}{\epsilon} \right) \frac{z^{1+\zeta} u'(C_{i1}(z))}{\gamma Y_t^{\zeta}} \right]^{\frac{\epsilon-1}{1+\zeta\epsilon}} \tag{55}$$

Since $C_{i1}(z)$ strictly increases on $m_{i1}(z)$ given P_1, then $\theta_{l1}(z) > \theta_{h1}(z)$.

3) Only the high-cash agent partially depletes for $z \in [\underline{z}, \overline{z}]$: Notice that, in this case:

$$\theta_{i1}(z) = \left[\left(\frac{\epsilon - 1}{\epsilon} \right) \frac{z}{\gamma} \beta \frac{P_1}{P_2} \frac{u'(C_{i2}(z))}{h_{i1}^{\zeta}}) \right]^{\epsilon-1}, \tag{56}$$

Assume by contradiction that $\theta_{l1}(z) \leq \theta_{h1}(z)$. This requires $p_{l1}(z) \geq p_{h1}(z)$ and, by (13), $u'(C_{h2}(z))/h_{h1}^{\zeta} \geq u'(C_{l2}(z))/h_{l1}^{\zeta}$. As shown before, for $i \in \{h, l\}$:

$$\frac{u'(C_{i2}(z))}{h_{i1}(z)^{\zeta}} = \left[\left(\frac{\epsilon}{\epsilon - 1} \right) \frac{\gamma}{\beta} \frac{P_2}{P_1} \right]^{\frac{\zeta\epsilon}{1+\zeta\epsilon}} Y_1^{\frac{-\zeta}{1+\zeta\epsilon}} z^{\frac{\zeta(1-\epsilon)}{1+\zeta\epsilon}} u'(C_{i2}(z))^{\frac{1}{1+\zeta\epsilon}}, \tag{57}$$

which, in turn, implies that $C_{l2}(z) \geq C_{h2}(z)$. Since the low-cash agent fully depletes, $R_{l1}(Z) \geq P_2 C_{l2}(z)$. Moreover, $\theta_{l1}(z) \leq \theta_{h1}(z)$ implies that $R_{h1}(z) \geq R_{l1}(z)$. However, since $s_{h1}(z) > 0$ and $s_{l1}(z) = 0$, $C_{l2}(z) \geq C_{h2}(z)$ cannot happen, since consumption at $t = 2$ is strictly increasing on $m_{i2}(z)$, given the price P_2. As a result, $\theta_{l1}(z) > \theta_{h1}(z)$.

I have proved above that $\theta_{l1}(z) \geq \theta_{h1}(z)$. This implies that $p_{l1}(z) \leq p_{h1}(z)$, $y_{l1}(z) \geq y_{h1}(z)$ and $R_{l1}(z) \geq R_{h1}(z)$ for all $z \in [\underline{z}, \overline{z}]$, where the inequalities hold strictly whenever the high-cash agent partially depletes. Now, I will prove, by an induction argument, that these results hold for any $t = \{1, 2, ...\}$ in which the economy is not in a stationary equilibrium. It suffices to show that, if the inequalities above hold for an arbitrary t, then $m_{h,t+1}(z) \geq m_{l,t+1}(z)$, implying that $\theta_{l,t+1}(z) \geq \theta_{h,t+1}(z)$.

Again, I will show it by cases. First, the case where both high- and low-cash agents fully deplete at t for some $z \in [\underline{z}, \overline{z}]$ is trivial, as it implies that $m_{l,t+1}(z) = m_{h,t+1}(z)$. Now, consider the other two cases — that is, either both types partially deplete at t or only the high-cash agent does. Assume, by contradiction, that $m_{lt}(z) < m_{ht}(z)$ and $m_{l,t+1}(z) > m_{h,t+1}(z)$. This means that $\theta_{lt}(z) > \theta_{ht}(z)$, and it requires $C_{l,t+1}(z) < C_{h,t+1}(z)$, which contradicts our assumption that $m_{l,t+1}(z) > m_{h,t+1}(z)$. I, therefore, conclude that $\theta_{lt}(z) \geq \theta_{ht}(z)$, $p_{lt}(z) \leq p_{ht}(z)$, $y_{lt}(z) \geq y_{ht}(z)$ and $R_{lt}(z) \geq R_{ht}(z)$ for all $z \in [\underline{z}, \overline{z}]$ and for all t such that the economy is not in a stationary equilibrium.

Characterization of inflation and relative revenues:
Integrating the individual price choices:

$$P_t = P_{t+1}\left(\frac{\epsilon}{\epsilon-1}\right)\frac{\gamma}{\beta}\left[\sum_{i\in\{h,l\}}\eta_i\int_{\underline{z}}^{\overline{z}}\frac{z^{\epsilon-1}u'(C_{i,t+1}(z))^{\epsilon-1}}{h_{it}(z)^{\zeta(\epsilon-1)}}dF_i(z)\right]^{\frac{1}{1-\epsilon}} \tag{58}$$

Now, notice that we can re-write:

$$\theta_{it}(z) = \left[\left(\frac{\epsilon-1}{\epsilon}\right)\frac{\beta}{\gamma}\frac{P_t}{P_{t+1}}\frac{zu'(C_{i,t+1}(z))}{h_{it}(z)^{\zeta}}\right]^{\epsilon-1}$$

This implies that $1+\pi_{t+1} = \overline{U}^{GAP}_{t+1}$ and $\frac{\theta_{it}(z)}{\theta_{i0}(z)} = \left(\frac{1}{1+\pi_{t+1}}\right)^{\epsilon-1} U^{GAP}_{i,t+1}(z)^{\epsilon-1} = \left(\frac{U^{GAP}_{i,t+1}(z)}{\overline{U}^{GAP}_{t+1}}\right)^{\epsilon-1}$.

Characterization of relative monetary holdings per income bracket:
Notice that, for any $z \in [\underline{z}, \overline{z}]$, the result above implies that $P_tC_{ht}(z) > P_tC_{lt}(z)$ and $R_{ht}(z) < R_{lt}(z)$ if the high-cash agent does not fully deplete, which means, by (3), that:

$$\Delta m_{h,t+1}(z) = R_{ht}(z) - P_tC_{ht}(z) < R_{lt}(z) - P_tC_{lt}(z) = \Delta m_{l,t+1}(z), \tag{59}$$

where $\Delta m_{i,t+1}(z) = m_{i,t+1}(z) - m_{it}(z)$ for $i \in \{h,l\}$.

The economy converges to the new stationary equilibrium in finite time:
Assume, by contradiction, that the mass of high-cash partially depleting agents remains forever bounded away from 0. Then, for these agents:

$$\lim_{t\to\infty}\frac{u'(C_{ht}(z))}{P_t} = \lim_{t\to\infty}\frac{u'(C_{h1}(z))}{\beta^{t-1}P_1} = \infty,$$

which implies that either $P_t \to 0$ or $C_{ht}(z) \to 0$. However, for partially depleting high-cash agents, $p_{ht}(z) > p_{lt}(z) \geq 0$, which implies that $P_t > 0$ for every $t = \{1, 2, ...\}$ by our contradiction assumption. This implies that $C_{ht} \to 0$, which cannot be the case either, since $m_{ht}(z) > m_{lt}(z) \geq 0$, which means that positive consumption is always available for these high-cash agents.

This proves that the mass of partially depleting agents goes to 0 in finite time, meaning that, at some time $T < \infty$, the economy reaches the new fundamental stationary equilibrium, in which the aggregate price is equal to $P_T = P^H(M_T) > 0$. The argument above ensures that no agent can partially deplete forever. Naturally, for any high-cash agent that partially depletes until $T-1$, it is the case that $m_{hT}(z) > m_{lT}(z)$, meaning that $C_{hT}(z) = m_{hT}(z)/P^H((1+\tau^A)M_0) > C_0(z)$. As a result, consumption only returns to the stationary equilibrium level for all agents at $T+1$. Finally, $Y_T = M_T/P^H(M_T) = Y_0$. □

A.3 Proposition 3

There is a non-negligible mass of fully depleting agents:

Now, I go on to prove that there is a non-negligible mass of fully depleting agents. Assume otherwise by means of contradiction. I begin by showing that, if there is no positive mass of fully depleting agents at t, then this must also be the case for $t+1$. Again, by contradiction, assume that there is some low-cash agent that fully depletes at $t+1$. This suffices as we have already shown that low-cash agents are more likely to fully deplete. As will be shown in more detail below, due to the homothety of preferences, $R_{lt}(z) = \overline{R_{lt}} z^{\frac{\epsilon-1}{\epsilon}} / \mathcal{Z}_l^{\frac{\epsilon-1}{\epsilon}}$ and $C_{lt}(z) = \overline{C_{lt}} z^{\frac{\epsilon-1}{\epsilon}} / \mathcal{Z}_l^{\frac{\epsilon-1}{\epsilon}}$, where $\overline{R_{lt}}$ and $\overline{C_{lt}}$ are respectively the average revenue and consumption among low-cash agents[22]. Thus:

$$(\overline{R_{lt}} - P_t \overline{C_{lt}}) \frac{z^{\frac{\epsilon-1}{\epsilon}}}{\mathcal{Z}_l^{\frac{\epsilon-1}{\epsilon}}} = R_{lt}(z) - P_t C_{lt}(z) = \Delta m_{l,t+1}(z)$$

Now, notice that, since $\Delta m_{i,t+1}(z)$ must aggregate to 0 for $i \in \{h, l\}$, and $\Delta m_{l,t+1}(z) > \Delta m_{h,t+1}(z)$ for every $z \in [\underline{z}, \overline{z}]$ according to Proposition 2, then $\Delta m_{l,t+1}(z) > 0$ for all $z \in [\underline{z}, \overline{z}]$. For the low-cash agent that partially depletes their money at t, but fully depletes at $t+1$, by the first-order condition, we have $P_{t+1} C_{l,t+1}(z) = \beta P_t C_{lt}(z) < m_{lt}(z) < m_{l,t+1}(z)$, a contradiction. Therefore, if there is no positive mass of partially depleting agents at t, then the same must hold for $t+1$ as well. This means that the economy will never converge to the fundamental stationary equilibrium, contradicting Proposition 2. Thus, there is a positive mass of fully depleting agents at any t.

Relative revenues obtained by low- and high-cash agents:

Since some agents need to fully deplete in equilibrium, and low-cash agents are more likely to do so, then some low-cash agents must choose full depletion. I will show that all low-cash agents must choose to fully deplete whenever any of them find it optimal. First, notice that, for any fully depleting low-cash agent, $p_{lt}(z)$ must satisfy (53). Therefore, for the low-cash agents:

$$\theta_{lt}(z) = \left[\left(\frac{\epsilon - 1}{\epsilon} \right) \frac{\beta}{\gamma} \frac{1}{Y_t^{1+\zeta}} \right]^{\frac{\epsilon-1}{\epsilon(1+\zeta)}} z^{\frac{\epsilon-1}{\epsilon}}, \tag{60}$$

[22] I will show later in this proof that this occurs both for fully and partially depleting agents, and it means that all agents of the same type, that is, either low- or high-cash, must make the same choice between full and partial depletion.

which is decreasing on aggregate output. Moreover, at $t = 1$, full depletion requires:

$$\frac{1}{\beta}\theta_{l1}(z)M_1^C = \frac{1}{\beta}R_{l1}(z) \geq m_{l1}(z) = (1 + \mathbb{1}_{\tau<0}\tau)\theta_{l0}(z)M_0$$
$$\frac{1}{\beta}Y_t^{\frac{1-\epsilon}{\epsilon}}M_t^C \geq (1 + \mathbb{1}_{\tau<0}\tau)Y_0^{\frac{1-\epsilon}{\epsilon}}M_0$$

where $\mathbb{1}_{\tau<0}$ takes the value of 1 for contractionary shock, and 0 otherwise. Notice that this condition does not depend on z, meaning that it holds for all low-cash agents. A simple induction argument generalizes this result for an arbitrary t. Now, I consider high-cash agents. The relative revenue for partially depleting agents is given by (55), which decreases on their current consumption. As before, homothety implies that all high-cash agents choose to save a positive amount whenever one of them does so. A similar argument to the one made above suffices to prove it, as the condition for partial depletion by high cash agents does not depend on z. I will now arrive at an expression for the consumption, at $t = 1$, of any high-cash agent as a function of the contemporaneous average high-cash consumption.

Average individual variables

Notice that the intertemporal budget constraint of any high-cash agent along the transition path can be written as:

$$P_1 C_{h1}(z)\sum_{t=1}^{T}\beta^{t-1} = m_{h1}(z) + \sum_{t=1}^{T-1}\theta_{ht}(z)P_t Y_t,$$

where I have imposed the first-order condition. Moreover, plugging (55) into it yields:

$$C_{h1}(z) = \left(\frac{1-\beta}{1-\beta^T}\right)\frac{1}{P_1}$$
$$\left\{(1 + \mathbb{1}_{\tau>0}\tau)m_0(z) + \left(\frac{z^{1+\zeta}}{C_{h1}(z)}\right)^{\frac{\epsilon-1}{1+\zeta\epsilon}}\left[\left(\frac{\epsilon-1}{\epsilon}\right)\frac{1}{\gamma P_1}\right]^{\frac{\epsilon-1}{1+\zeta\epsilon}}\sum_{t=1}^{T-1}\beta^{\frac{(t-1)(1-\epsilon)}{1+\zeta\epsilon}}P_t^{\frac{\epsilon(1+\zeta)}{1+\zeta\epsilon}}Y_t^{\frac{1+\zeta}{1+\zeta\epsilon}}\right\},$$

which, using (22), can be simplified to:

$$C_{h1}(z) = Az^{\frac{\epsilon-1}{\epsilon}} + Bz^{\frac{(1+\zeta)(\epsilon-1)}{1+\zeta\epsilon}}C_{h1}(z)^{\frac{1-\epsilon}{1+\zeta\epsilon}},$$

where A and B are common across all high-cash agents. Now, one can guess that $C_{h1}(z) = \overline{C_{h1}}z^D \big/ \mathcal{Z}_h^D$, where $\overline{C_{h1}}$ is the average consumption by high-cash agents, and D is a constant. By plugging this in the equation above, we obtain $D = \frac{\epsilon-1}{\epsilon}$. By the first-order condition of partially depleting agents, this means that the whole consumption path is the same across high-cash agents as well as the moment where they decide to fully deplete, T. Together with (60), this implies that $X_{i1}(z) = \overline{X_{i1}}\frac{z^{\frac{\epsilon-1}{\epsilon}}}{\mathcal{Z}_i^{\frac{\epsilon-1}{\epsilon}}}$ and $\overline{X_{i1}} = X_{i1}(\mathcal{Z}_i)$ for $X \in \{C, \theta, m\}$, $i \in \{h, l\}$ and $z \in [\underline{z}, \overline{z}]$.

Output and consumption paths:

The results above imply that $\theta_{lt}(z) > \theta_0(z) > \theta_{ht}(z)$ for every $z \in [\underline{z}, \overline{z}]$ and $t = \{1, ..., T-1\}$. Equations (60) and (55) imply, respectively, that $Y_t < Y_0$ and $C_{ht}(z) > C_0(z) > C_{lt}(z)$ for every $z \in [\underline{z}, \overline{z}]$ and t along the transition, where the latter inequality relies on the former. Moreover, notice that, by (13):

$$p_{it}(z) = \left[\left(\frac{\epsilon}{\epsilon - 1} \right) \frac{\gamma}{z^{1+\zeta}\beta} P_t^{\zeta\epsilon} Y_t^{\zeta} P_{t+1} C_{i,t+1}(z) \right]^{\frac{1}{1+\zeta\epsilon}} \tag{61}$$

for $z \in [\underline{z}, \overline{z}]$ and $i \in \{h, l\}$. Aggregating it, and computing the corresponding $\theta_{it}(z)$ gives:

$$\theta_{it}(z) = \frac{\left(\frac{z^{1+\zeta}}{P_{t+1}C_{i,t+1}(z)} \right)^{\frac{\epsilon-1}{1+\zeta\epsilon}}}{\sum_{j\in\{h,l\}} \eta_j \int_{\underline{z}}^{\overline{z}} \left(\frac{\hat{z}^{1+\zeta}}{P_{t+1}C_{j,t+1}(\hat{z})} \right)^{\frac{\epsilon-1}{1+\zeta\epsilon}} dF_j(\hat{z})},$$

which means that, for any $z \in [\underline{z}, \overline{z}]$:

$$\frac{\theta_{lt}(z)}{\theta_{ht}(z)} = \left(\frac{P_{t+1}C_{h,t+1}(z)}{P_{t+1}C_{l,t+1}(z)} \right)^{\frac{\epsilon-1}{1+\zeta\epsilon}} \tag{62}$$

I already argued that, given the homothety of preferences, we must have $m_{l,t+1}(z) > m_{lt}(z)$ and $m_{h,t+1}(z) < m_{ht}(z)$ for every $z \in [\underline{z}, \overline{z}]$. This means that, for low-cash agents, $P_t C_{lt}(z) = m_{lt}(z)$ grows over time. For high-cash agents, on the other hand, $P_t C_{lt}(z) = \beta P_{t-1} C_{h,t-1}(z)$ for $t \in \{2, 3, ..., T-1\}$, meaning that it decreases over time. Thus, $\theta_{l0}(z)/\theta_{h0}(z) \leq \theta_{l,t+1}(z)/\theta_{h,t+1}(z) < \theta_{lt}(z)/\theta_{ht}(z)$ for $t \in \{1, 2, ..., T-1\}$. Due to the homothety of preferences and the fact that $\theta_{it}(z)$ must integrate to 1, this means that $\theta_0(z) \leq \theta_{l,t+1}(z) < \theta_{lt}(z)$ and $\theta_0(z) \geq \theta_{h,t+1}(z) > \theta_{ht}(z)$. By (60), this implies that $Y_0 \geq Y_{t+1} > Y_t$ with strict inequality for $t = \{1, ..., T-1\}$ and equality for $t = T$. Finally, with this in mind, (55) implies that $C_0(z) \leq C_{h,t+1}(z) < C_{ht}(z)$.

Characterization of T:

Notice that $P_T C_{hT}(z) = \beta^{T-1} P_1 C_{h1}(z)$ for $z \in [\underline{z}, \overline{z}]$ by the first-order condition and budget constraint of high-cash agents. Given that, as proven in Proposition 2, $C_{hT}(z) > \theta_0(z)(1+\tau^A)M_0/P_T$, we obtain the condition:

$$\theta_0(z)(1+\tau^A)M_0 < \beta^{T-1} P_1 C_{h1}(z) < \beta^{T-1}\theta_0(z)(1+\mathbb{1}_{\tau>0}\tau)M_0$$

$$\beta^{T-1} > \frac{1+\tau^A}{1+\mathbb{1}_{\tau>0}\tau} \tag{63}$$

Let the maximum value of T that satisfies it be denoted by T^{MAX}. Then, $T \leq T^{MAX}$.

Characterization of prices:

Since low-cash agents always fully deplete their resources, (53) implies that:

$$y_{lt}(z) = z\left[\left(\frac{\epsilon-1}{\epsilon}\right)\frac{\beta}{\gamma}\right]^{\frac{1}{1+\zeta}}$$

meaning that $h_{l,t+1}(z) = h_{lt}(z) = h_{lt}(z')$ for all periods and any $z, z' \in [\underline{z}, \overline{z}]$. Moreover:

$$P_{t+1}C_{l,t+1}(z) = R_{lt}(z) = \theta_{lt}(z)M_t^C > \theta_0(z)M_t^C$$

which means that $p_{ht}(z) > p_{lt}(z) > p^H(z, M_t^C)$ by (13) and the stability of $h_{lt}(z)$. This implies that: (i) $P(M_t^C) > P^H(M_t^C)$; and (ii) low cash agents' prices are proportional to $p_{lt}(z)y_{lt}(z) = m_{l,t+1}(z)$. As a result, $p_{lt}(z) \leq p_{l,t+1}(z)$, with strict inequality for $t \in \{1, 2, ..., T-1\}$.

Lastly, by an argument already made in the proof of Proposition 1, one can show that:

$$\left[\sum_{i\in\{h,l\}}\eta_i\int_{\underline{z}}^{\overline{z}}\frac{z^{\frac{\epsilon-1}{1+\zeta\epsilon}}}{C_{it}(z)^{\frac{\epsilon-1}{1+\zeta\epsilon}}}dF_i(z)\right]^{\frac{1+\zeta\epsilon}{\epsilon-1}} > \left(\frac{\epsilon}{\epsilon-1}\right)\gamma Y_t^{\zeta}$$

for every $t \in \{1, 2, ...\}$, where the strong inequality follows from the fact that, by the first-order condition of low-cash agents, $C_{l,t+1}(z) > \beta\frac{P_t}{P_{t+1}}C_{lt}(z)$. We can plug this into the aggregate price expression (47), which gives us:

$$\frac{P_{t+1}}{P_t} > \beta\left(\frac{Y_{t+1}}{Y_t}\right)^{\zeta} \geq \beta, \tag{64}$$

where the last inequality holds strictly for $t \in \{1, 2, ..., T-1\}$, since $Y_{t+1} \geq Y_t$ then. □

A.4 Proposition 4

Lower interest rate, $1 \geq q_t > \beta$:

I will show that $T = \infty$ is not possible through a simple proof by contradiction. Assume $T = \infty$. As the borrowing constraint only binds to low-cash agents, we have:

$$\frac{u'(C_{ht}(z))}{P_t} = \frac{\beta}{q_t}\frac{u'(C_{h,t+1}(z))}{P_{t+1}} \qquad \text{and} \qquad \frac{u'(C_{lt}(z))}{P_t} > \frac{\beta}{q_t}\frac{u'(C_{l,t+1}(z))}{P_{t+1}} \tag{65}$$

which hold for $t \in \{1, 2, ...\}$ if $q_t > \beta$. Thus, (47) can be re-written as $P_{t+1} > \left(\frac{\beta}{q_{t+1}}\right)^{\frac{1}{1+\zeta}} P_t$. Since $q_t = q_{t+1}$, we have $u'(C_{ht}(z)) < \left(\frac{\beta}{q_t}\right)^{\frac{\zeta}{1+\zeta}} u'(C_{h,t+1}(z)) < u'(C_{h,t+1}(z))$ by (65). This means that $C_{h,t+1}(z) < C_{ht}(z)$ for all $t \in \{1, 2, ...\}$.

I will now show that $\lim_{t\to\infty} C_{ht}(z) = 0$ for all high-cash agents. Assume, by contradiction, that there is some high-cash agent with productivity $z \in [\underline{z}, \overline{z}]$ whose consumption does not converge to 0. Let $\varepsilon_{h,t+1}(z) := u'(C_{h,t+1}(z)) - u'(C_{ht}(z))$ and $U_{hT}^{MAX}(z) := \lim_{t\to\infty} u'(C_{ht}(z)) < \infty$. Naturally, since marginal utility converges to a constant, $\lim_{t\to\infty} \varepsilon_{h,t+1}(z) = 0$. By (65), we must have:

$$\lim_{t\to\infty} \frac{P_{t+1}}{P_t} = \lim_{t\to\infty} \frac{\beta}{q_t}\left(1 + \frac{\varepsilon_{h,t+1}(z)}{u'(C_{ht}(z))}\right) = \frac{\beta}{q_t},$$

which uses the fact that q_t is assumed to be constant. However, this implies that $\frac{\beta}{q_t} = \lim_{t\to\infty} \frac{P_{t+1}}{P_t} > \left(\frac{\beta}{q_t}\right)^{\frac{1}{1+\zeta}}$, implying that $\beta > q_t$, a contradiction. This means that $\lim_{t\to\infty} C_{ht}(z) = 0$ for every $z \in [\underline{z}, \overline{z}]$. However, since $C_{ht}(z) > C_{lt}(z) \geq 0$ for as long as high-cash agents do not fully deplete, this cannot occur. This proves that $T < \infty$. Since (65) is never satisfied with equality for the low cash agent for $q_t > \beta$, then the economy must achieve the fundamental stationary equilibrium.

Higher interest rate, $q_t < \beta$:

This case is straightforward, as we can make a similar argument as the one above. In this case, there is excess bond supply, meaning that:

$$\frac{u'(C_{ht}(z))}{P_t} < \frac{\beta}{q_t}\frac{u'(C_{h,t+1}(z))}{P_{t+1}} \qquad \text{and} \qquad \frac{u'(C_{lt}(z))}{P_t} = \frac{\beta}{q_t}\frac{u'(C_{l,t+1}(z))}{P_{t+1}}$$

As $q_{t+1} = q_t < \beta$, (47) now implies that $P_{t+1} < \left(\frac{\beta}{q_{t+1}}\right)^{\frac{1}{1+\zeta}} P_t$. A similar reasoning as the one above shows that $C_{l,t+1}(z) > C_{lt}(z)$ for $z \in [\underline{z}, \overline{z}]$ and $t \in \{1, 2, ...\}$, meaning that $C_{h,t+1}(z) < C_{ht}(z)$ for a non-negligible mass of high-cash agents. We can, again, prove by contradiction that $\lim_{t\to\infty} C_{ht}(z) = 0$, since assuming otherwise implies that $\frac{\beta}{q_t} = \lim_{t\to\infty} \frac{P_{t+1}}{P_t} < \left(\frac{\beta}{q_t}\right)^{\frac{1}{1+\zeta}}$, and, hence, $\beta < q_t$. Again, $C_{ht}(z) > C_{lt}(z) \geq 0$ proves that these high-cash agents' consumption cannot go to zero and, hence, $T < \infty$. $\square$

A.5 Proposition 5

Throughout this proof, I will use the homothety of preferences to facilitate the arguments, meaning that, if any low-cash agent is constrained, all of them are. Moreover, choices made by any high-cash (low-cash) agent are proportional to the choices made by all the other high-cash (low-cash). Notice that the result holds trivially in the case where (37) does not bind from the beginning, as the economy is already in the non-fundamental stationary equilibrium. Thus, I will focus on the case of economies that start off with constrained low-cash agents in period $T = 1$. In this case, (65) also holds.

Notice that, since higher revenues may relax the borrowing constraint, the relative revenue of low-cash agents is now given by:

$$\theta_{lt}(z) = \left[\left(\frac{\epsilon-1}{\epsilon}\right)\frac{z^{1+\zeta}}{\gamma}\frac{P_t}{Y_t^{\zeta}}\left(\frac{\beta}{P_{t+1}C_{l,t+1}(z)}+\kappa\frac{\Phi_{lt}(z)}{\beta^t}\right)\right]^{\frac{\epsilon-1}{1+\zeta\epsilon}}$$

where $\Phi_{lt}(z) := \beta^t\left(\frac{q_t}{P_tC_{lt}(z)} - \frac{\beta}{P_{t+1}C_{l,t+1}(z)}\right)$ is the multiplier of the collateral constraint (37). Thus, we can write:

$$\begin{aligned}\frac{\theta_{lt}(z)}{\theta_{ht}(z)} &= \left[\frac{P_{t+1}C_{h,t+1}(z)}{\beta}\left(\frac{\beta}{P_{t+1}C_{l,t+1}(z)}+\kappa\frac{\Phi_{lt}(z)}{\beta^t}\right)\right]^{\frac{\epsilon-1}{1+\zeta\epsilon}} \\ &< \left[\frac{P_{t+1}C_{h,t+1}(z)}{\beta}\left(\frac{\beta}{P_{t+1}C_{l,t+1}(z)}+\frac{\Phi_{lt}(z)}{\beta^t}\right)\right]^{\frac{\epsilon-1}{1+\zeta\epsilon}} = \left[\frac{P_tC_{ht}(z)}{P_tC_{lt}(z)}\right]^{\frac{\epsilon-1}{1+\zeta\epsilon}},\end{aligned} \tag{66}$$

where the equality follows from the definition $\Phi_{lt}(z)$ and from (65). Moreover, using (66) again, since $\Phi_{lt}(z) > 0$, we have:

$$\frac{\theta_{lt}(z)}{\theta_{ht}(z)} > \left[\frac{P_{t+1}C_{h,t+1}(z)}{P_{t+1}C_{l,t+1}(z)}\right]^{\frac{\epsilon-1}{1+\zeta\epsilon}} \geq \frac{\theta_{l,t+1}(z)}{\theta_{h,t+1}(z)}, \tag{67}$$

where the second inequality holds with equality if the economy attains a stationary equilibrium at $t+1$[23]. Moreover, plugging (2) into (3), and using (37), yields:

$$m_{l,t+1}(z) = [1-\kappa]R_{lt}(z), \tag{68}$$

whenever the low-cash agent is constrained in period t.

Furthermore, this economy might be stuck in the zero lower bound (ZLB). To see this, notice that, given the optimal consumption expenditure path $\{P_tC_{ht}(z)\}^{\infty}$ for all $z \in [\underline{z}, \overline{z}]$, if $\eta_h \int_{\underline{z}}^{\overline{z}} \frac{m_{ht}(z)-P_tC_{ht}(z)}{q_t} dF_h(z) > (1-\eta_h)\kappa \int_{\underline{z}}^{\overline{z}} R_{lt}(z)dF_l(z)$ for any $q_t < 1$ — meaning that there is excess demand for bonds above the ZLB, — then we must have $q_t = 1$ and positive cash savings, *i.e.*, $s_{h,t+1}(z) > 0$. The remainder of the proof will be divided into two cases: 1) $q_t < 1$ and 2) $q_t = 1$.

Out of the zero lower bound, *i.e.*, $q_t < 1$:

Naturally, cash savings are $s_{h,t+1}(z) = 0$ for all $z \in [\underline{z}, \overline{z}]$, meaning that $M_t^C = M_t$. Assume, by contradiction, that $q_t \leq \beta$ for any $t \in \{1, 2, ...\}$ in which all low-cash agents are constrained. By (65), our contradiction assumption implies that $P_{t+1}C_{h,t+1}(z) \geq P_tC_{ht}(z)$ for $z \in [\underline{z}, \overline{z}]$. Thus, $P_{t+1}C_{l,t+1}(z) \leq P_tC_{lt}(z)$ to ensure that expenditures sum

[23] In this case, $\Phi_{lt}(z) = 0$ and either $P_{t+2}C_{i,t+2}(z) = \beta P_{t+1}C_{i,t+1}(z)/q_{t+1}$ or $P_{t+2}C_{i,t+2}(z) = P_{t+1}C_{i,t+1}(z)$ in the cases of, respectively, a non-fundamental and a fundamental stationary equilibrium.

up to $M_{t+1} \leq M_t$[24]. However, by (65), $P_{t+1}C_{l,t+1}(z) > P_tC_{lt}(z)$ for $q_t \leq \beta$, a contradiction. This implies that $1 > q_t > \beta$ while low-cash agents are constrained. Equation (67) implies that $R_{l,t+1}(z) < R_{lt}(z)$ and $R_{h,t+1}(z) > R_{ht}(z)$.

Now, I will show that this economy must converge to a stationary equilibrium. Again, I will make an argument by contradiction. Assume that convergence does not happen in finite time. Thus, for any $t \in \{1, 2, ...\}$, $q_t > \beta$ and $b_{l,t+1}(z) > 0$ for $z \in [\underline{z}, \overline{z}]$. This implies, by (65), that $P_{t+1}C_{h,t+1}(z) = \frac{\beta}{q_t}P_tC_{ht}(z) < P_tC_{ht}(z)$. Let us define $\vartheta_{ht}(z) := P_tC_{ht}(z) - P_{t+1}C_{h,t+1}(z)$. As proven before, consumption expenditures by high-cash agents cannot collapse to zero, so $\lim_{t\to\infty} \vartheta_{ht}(z) > 0$ can be ruled out. I only need to prove that $\lim_{t\to\infty} \vartheta_{ht}(z) = 0$ cannot occur. Assume it does by contradiction. Then, the economy converges to a stationary equilibrium asymptotically.

Assume by contradiction that 1) the low-cash agents are still constrained in $t + 1$, and 2) that the economy does not, at any point, go to the ZLB. Then, since $R_{l,t+1}(z) < R_{lt}(z)$, (68) implies that $m_{l,t+2}(z) < m_{l,t+1}(z)$. Since $P_{t+2}C_{l,t+2}(z) > P_{t+1}C_{l,t+1}(z)$, we must have $q_{t+2}b_{l,t+3}(z) < q_{t+1}b_{l,t+2}(z)$ by (2). However, by (37), $b_{l,t+3}(z) \geq -\kappa R_{l,t+2}(z) > -\kappa R_{l,t+1}(z) = b_{l,t+2}(z)$. This immediately implies that $1 \geq q_{t+2} > q_{t+1} > \beta$, meaning that low-cash agents are also constrained at $t + 2$.

By induction, this economy must remain constrained forever and, $\lim_{t\to\infty} q_t > \beta$. This means that this economy does not converge to a non-fundamental stationary equilibrium. Furthermore, the fact that $m_{l,t+1}(z) < m_{lt}(z)$ implies that $\lim_{t\to\infty} m_{lt}(z) < \lim_{t\to\infty} m_{ht}(z)$, meaning that this economy does not converge to a fundamental stationary equilibrium either. This contradicts the fact that $\lim_{t\to\infty} \vartheta_{ht}(z) = 0$, proving that the economy either returns to the stationary equilibrium at $t + 1$ or that it eventually achieves the ZLB in finite time.

To rule out the possibility of the economy achieving the ZLB, I will show that low-cash agents are less constrained at $t + 1$ than at t. To begin, consider an economy with a nominal interest rate that is fixed at q_t and with the same initial monetary holdings distribution. I will denote it with the superscript FI — for "fixed interest rate". To begin, I will show that $m^{FI}_{l,t+1}(z) > m^{FI}_{lt}(z)$ for every t and $z \in [\underline{z}, \overline{z}]$ by means of a backwards induction argument. As shown in Proposition 4, there is a time T in which $P^{FI}_T C^{FI}_{hT}(z) = m^{FI}_{hT}(z)$ and, hence, $m^{FI}_{h,T+1}(z) < m^{FI}_{hT}(z)$ and $m^{FI}_{l,T+1}(z) > m^{FI}_{lT}(z)$. Assume that, for an arbitrary $t \in \{2, 3, ..., T - 1\}$ in the transition path, $m^{FI}_{h,t+1}(z) < m^{FI}_{ht}(z)$ and $m^{FI}_{l,t+1}(z) > m^{FI}_{lt}(z)$.

[24]Here, I am allowing the economy to go the ZLB in the next period, in which case, $M_{t+1} < M_t$

By contradiction, assume that $m^{FI}_{ht}(z) \geq m^{FI}_{h,t-1}(z)$. In this economy, the borrowing constraint cannot be relaxed by setting a lower price. As a result:

$$\frac{\theta^{FI}_{lt}(z)}{\theta^{FI}_{ht}(z)} = \left[\frac{P^{FI}_{t+1}C^{FI}_{h,t+1}(z)}{P^{FI}_{t+1}C^{FI}_{l,t+1}(z)}\right]^{\frac{\epsilon-1}{1+\zeta\epsilon}}$$

Since, by (65), we must have $P^{FI}_t C^{FI}_{ht}(z) < P^{FI}_{t-1}C^{FI}_{h,t-1}(z)$ and, since $M^{C,FI}_t = M_t = M^{C,FI}_{t-1}$, $P^{FI}_t C^{FI}_{lt}(z) > P^{FI}_{t-1}C^{FI}_{l,t-1}(z)$. This immediately implies that $\frac{\theta^{FI}_{lt}(z)}{\theta^{FI}_{ht}(z)} < \frac{\theta^{FI}_{l,t-1}(z)}{\theta^{FI}_{h,t-1}(z)}$ and, thus, $R^{FI}_{ht}(z) > R^{FI}_{h,t-1}(z)$. By our contradiction assumption and $P^{FI}_t C^{FI}_{ht}(z) < P^{FI}_{t-1}C^{FI}_{h,t-1}(z)$, (2) implies that $b^{FI}_{h,t+1}(z) > b^{FI}_{ht}(z)$. By (3), this means that:

$$\begin{aligned}0 > m^{FI}_{h,t+1}(z) - m^{FI}_{ht}(z) &= R^{FI}_{ht}(z) + (1-q_t)b^{FI}_{h,t+1}(z) - P^{FI}_t C^{FI}_{ht}(z)\\ &> R^{FI}_{h,t-1}(z) + (1-q_{t-1})b^{FI}_{ht}(z) - P^{FI}_{t-1}C^{FI}_{h,t-1}(z)\\ &= m^{FI}_{ht}(z) - m^{FI}_{h,t-1}(z),\end{aligned}$$

where we have used the fact that $q_t = q_{t-1}$ in the fixed interest rate economy. This is a contradiction, which proves that $m^{FI}_{h,t+1}(z) < m^{FI}_{ht}(z)$ and $m^{FI}_{l,t+1}(z) > m^{FI}_{lt}(z)$ for an arbitrary t. Now, assume by contradiction that $m_{l,t+1}(z) \leq m_{lt}(z)$ in the constrained economy. Then, $P_{t+1}C_{l,t+1}(z) < P^{FI}_{t+1}C^{FI}_{l,t+1}(z)$, meaning that $P_{t+1}C_{h,t+1}(z) > P^{FI}_{t+1}C^{FI}_{h,t+1}(z)$. However, this means that:

$$\frac{\theta^{FI}_{lt}(z)}{\theta^{FI}_{ht}(z)} < \left[\frac{P_{t+1}C_{h,t+1}(z)}{P_{t+1}C_{l,t+1}(z)}\right]^{\frac{\epsilon-1}{1+\zeta\epsilon}} < \frac{\theta_{lt}(z)}{\theta_{ht}(z)}.$$

Since $M^C_t = M_t = M^{C,FI}_t$, this implies that $R_{lt}(z) > R^{FI}_{lt}(z)$. Moreover, notice that we must also have $\frac{\beta}{q_t}P_t C_{ht}(z) > \frac{\beta}{q_t}P^{FI}_t C^{FI}_{ht}(z)$. Since monetary holdings are the same at t in both economies, then $q_t b_{h,t+1}(z) < q_t b^{FI}_{h,t+1}(z)$ and, thus, $b_{l,t+1}(z) > b^{FI}_{l,t+1}(z)$. However, this means that $m_{l,t+1}(z) = R_{lt}(z) + b_{l,t+1}(z) > R^{FI}_{lt}(z) + b^{FI}_{l,t+1}(z) = m^{FI}_{l,t+1}(z) > m_{lt}(z)$, a contradiction. This proves that $m_{l,t+1}(z) > m_{lt}(z)$, implying that low-cash agents cannot become more constrained at $t+1$, and, therefore, the economy cannot go to the ZLB. As a result, the stationary equilibrium must be attained at $t+1$.

Zero lower bound, *i.e.*, $q_t = 1$:

If the economy remains in the ZLB forever, by Proposition 2 and Proposition 4, it must eventually return to a fundamental stationary equilibrium. In this case, we must have $b_{l,T+1}(z) = 0$, optimal (unrealized) bond sales satisfying $b^*_{l,T+1}(z) < -\kappa R^*_{lT}(z)$, and $\frac{\beta}{q_T}m_{hT}(z) \leq \theta_0(z)M_{T+1}$ for $z \in [\underline{z}, \overline{z}]$. However, it can go to an equilibrium with $1 > q_t \geq \beta$ before that. In fact, there must be a period T' where $s_{h,T'+1}(z) = 0\ \forall z \in [\underline{z}, \overline{z}]$. If (37) still binds then, we must have $1 > q_{T'+1} > \beta$; and, if it does not, we have $q_{T'+1} = \beta$.

Non-fundamental stationary equilibrium:

If a non-fundamental stationary equilibrium is achieved at T, then, for high-cash agents:

$$\theta_0(z)M_{T+1} < P_{T+1}C_{h,T+1}(z) = \frac{\beta}{q_T}P_T C_{hT}(z) < \frac{\beta}{q_T}m_{hT}(z) = m_{hT}(z),$$

where the first inequality comes from the fact that high-cash consumption expenditure must be higher than their monetary holdings at the fundamental stationary equilibrium. Notice that the expression above relies on the homothety of the preferences since all high-cash agents decide to fully deplete at the same time. However, $m_{hT}(z) > \theta_0(z)M_{T+1}$ is trivially satisfied. So, the economy achieves a non-fundamental stationary equilibrium at T if, and only if, the low-cash optimal bond sales satisfies $b^*_{l,T+1}(z) \geq -\kappa R^*_{lT}(z)$. By (30) and (31), this requires $m_{lT}(z) \geq (1-\kappa)R^*_{lT}(z)$ for $z \in [\underline{z}, \overline{z}]$.

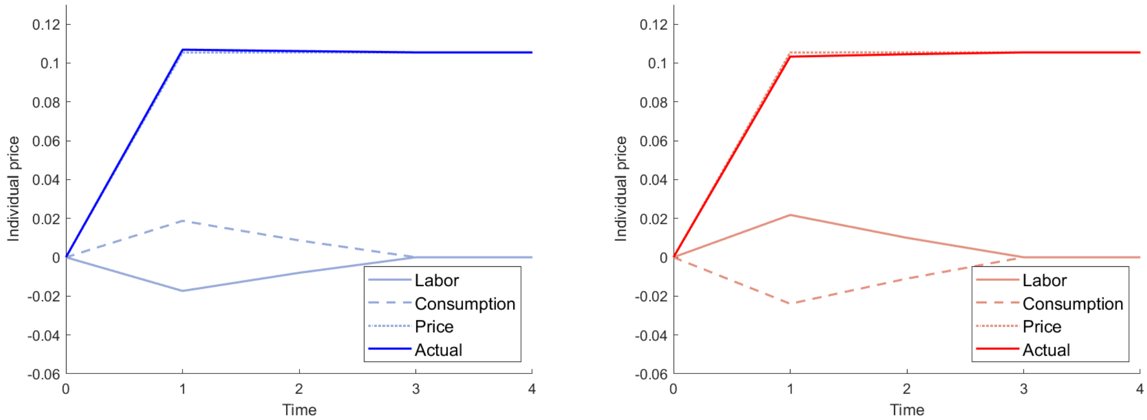
Fundamental stationary equilibrium:

The economy goes to the fundamental stationary equilibrium at T if high-cash agents are not willing to buy bonds. They decide to fully deplete if:

$$\theta_0(z)M_{T+1} = P_{T+1}C_{h,T+1}(z) \geq \frac{\beta}{q_T}m_{hT}(z) > \frac{\beta}{q_T}\theta_0(z)M_{T+1}.$$

Notice that it can only occur if $q_T > \beta$. Even though no borrowing occurs at T, this would happen if the optimal bond sales could not be achieved, that is, $b^*_{l,T+1}(z) < -\kappa R^*_{lT}(z)$ and, hence, $m_{lT}(z) < (1-\kappa)R^*_{lT}(z)$ for $z \in [\underline{z}, \overline{z}]$. $\square$

B Outstanding Graphs



(a) Connected agents' prices, $p_{ct}(\mathcal{Z})$ (b) Unconnected agents' prices, $p_{ut}(\mathcal{Z})$

Figure 6: Decomposition of individual prices in the full enforcement economy

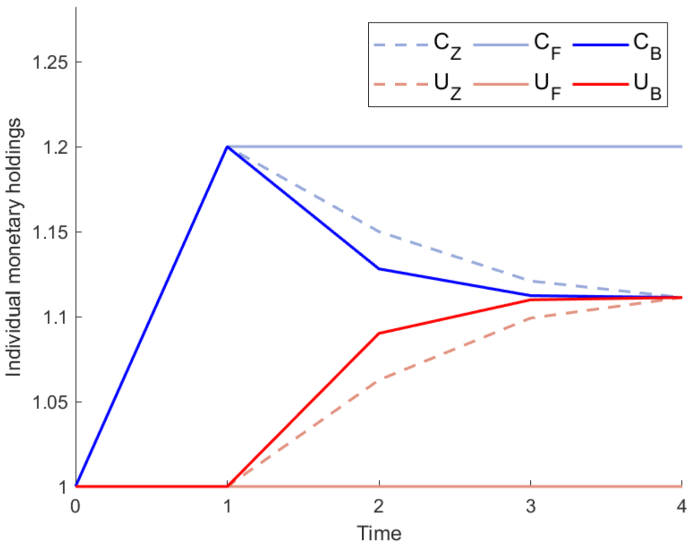
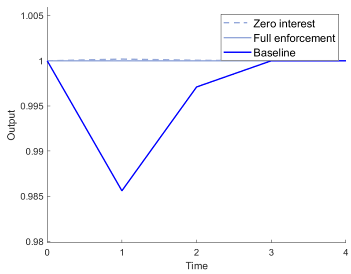
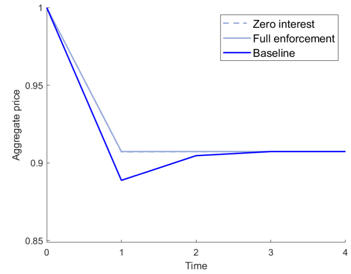


Figure 7: Comparison of monetary holdings, m_t, across agents

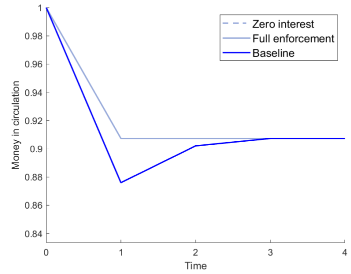
Notes: Connected (C) and unconnected (U) cases. Subscripts denote: baseline (B), full enforcement (F), and zero interest rate (Z).



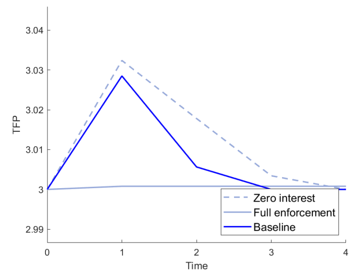
(a) Path of output, Y_t



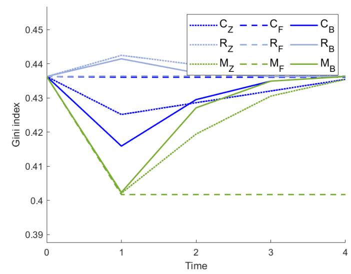
(b) Path of price, P_t



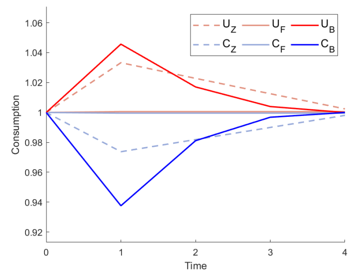
(c) Money in circul., M_t^C



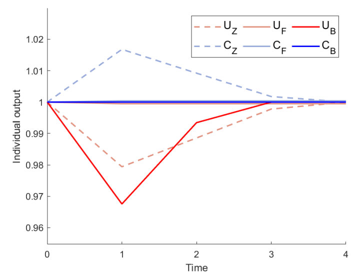
(d) Total Factor Productivity



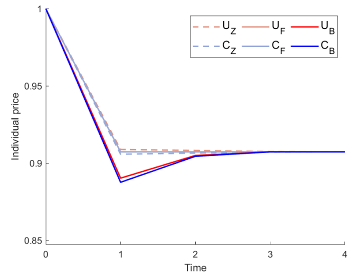
(e) Gini indexes



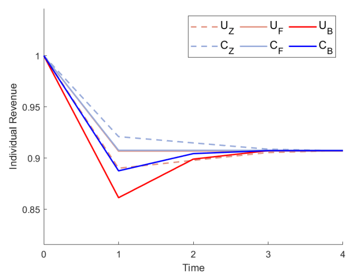
(f) Indiv. consumption, $C_{it}(z)$



(g) Individual output, $y_{it}(z)$



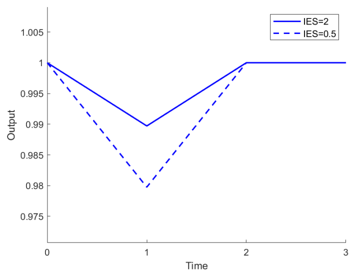
(h) Individual prices, $p_{it}(z)$



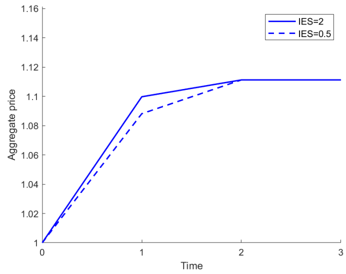
(i) Indiv. revenues, $R_{it}(z)$

Figure 8: Paths for aggregate and individual variables under a negative shock

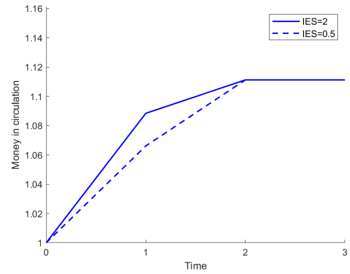
Notes: Connected (C) and unconnected (U) cases. Subscripts denote: baseline (B), full enforcement (F), and zero interest rate (Z). For the Gini indexes, $G \in \{C, R, M\}$ stand for consumption, revenue, and money holdings.



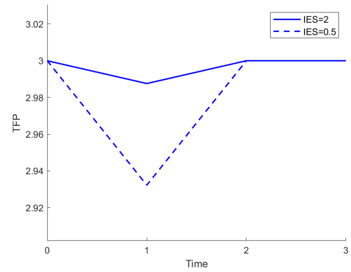
(a) Path of output, Y_t



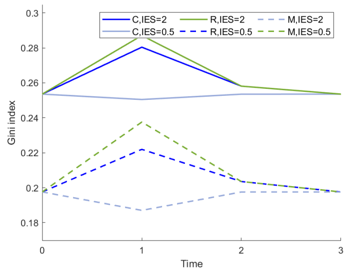
(b) Path of price, P_t



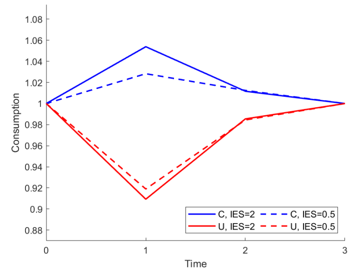
(c) Money in circul., M_t^C



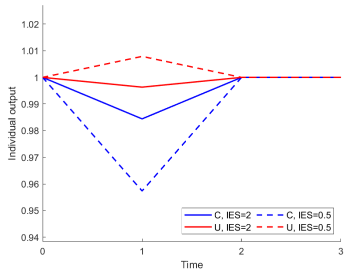
(d) Total Factor Productivity



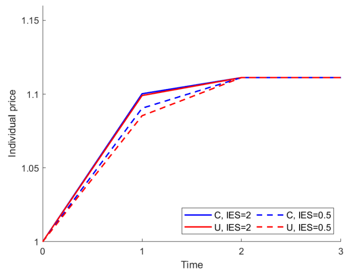
(e) Gini indexes



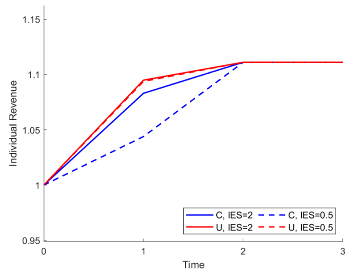
(f) Indiv. consumption, $C_{it}(z)$



(g) Individual output, $y_{it}(z)$



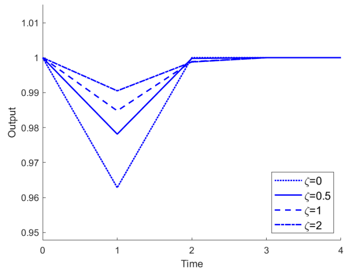
(h) Individual prices, $p_{it}(z)$



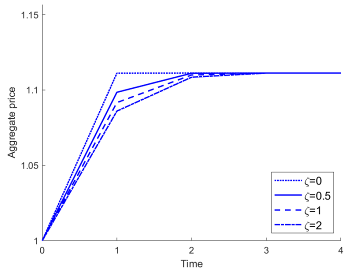
(i) Indiv. revenues, $R_{it}(z)$

Figure 9: Paths for aggregate and individual variables under CRRA utility

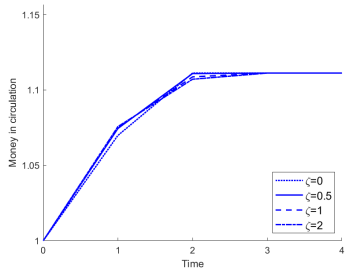
Notes: Connected (C) and unconnected (U) cases. For the Gini indexes, $G \in \{C, R, M\}$ stand for consumption, revenue, and money holdings.



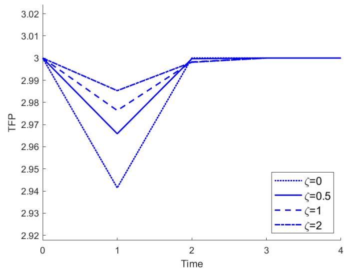
(a) Path of output, Y_t



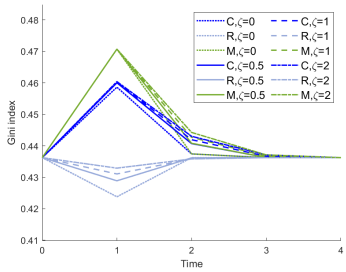
(b) Aggregate price, P_t



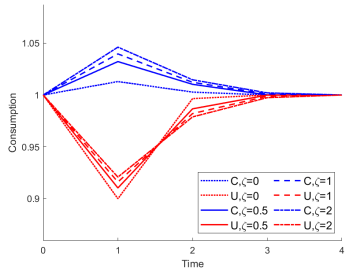
(c) Money in circulation, M_t^C



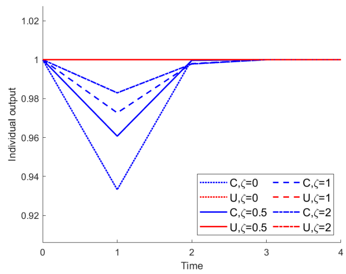
(d) Total Factor Productivity



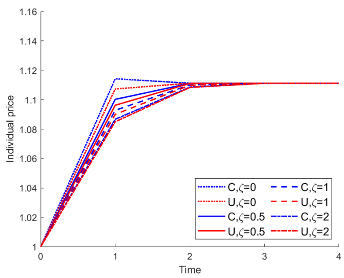
(e) Gini indexes



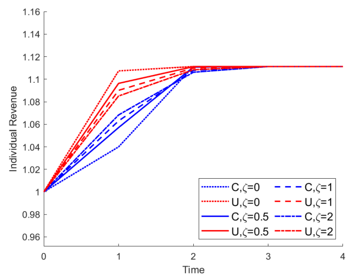
(f) Indiv. consumption, $C_{it}(\mathcal{Z})$



(g) Individual output, $y_{it}(\mathcal{Z})$



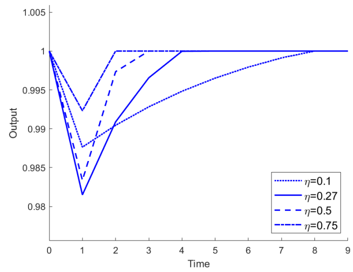
(h) Individual prices, $p_{it}(\mathcal{Z})$



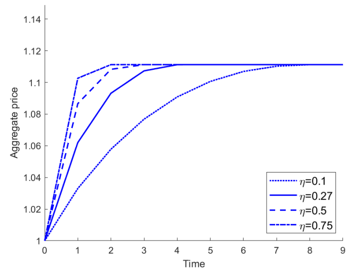
(i) Indiv. revenues, $R_{it}(\mathcal{Z})$

Figure 10: Paths for aggregate and individual variables under different Frisch elasticities, $\frac{1}{\xi}$

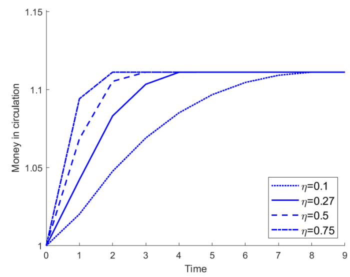
Notes: Connected (C) and unconnected (U) cases. For the Gini indexes, $G \in \{C, R, M\}$ stand for consumption, revenue, and money holdings.



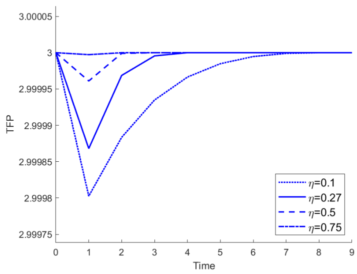
(a) Path of output, Y_t



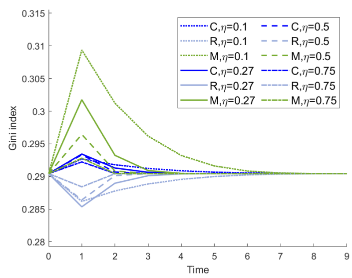
(b) Aggregate price, P_t



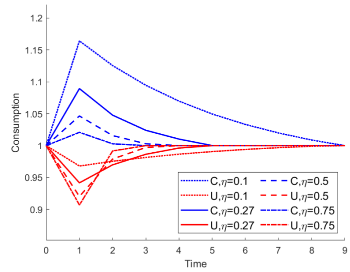
(c) Money in circulation, M_t^C



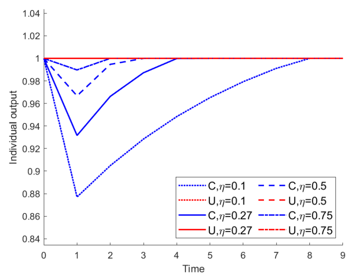
(d) Total Factor Productivity



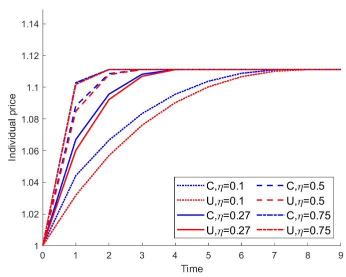
(e) Gini indexes



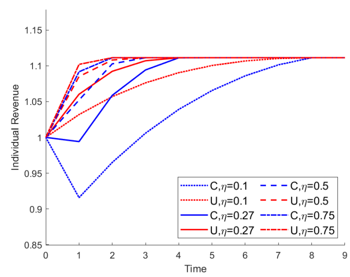
(f) Indiv. consumption, $C_{it}(\mathcal{Z})$



(g) Individual output, $y_{it}(\mathcal{Z})$



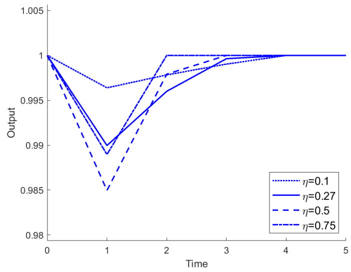
(h) Individual prices, $p_{it}(\mathcal{Z})$



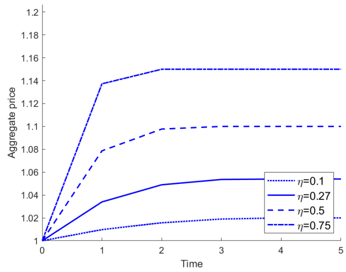
(i) Indiv. revenues, $R_{it}(\mathcal{Z})$

Figure 11: Paths for aggregate and individual variables for different values of η and τ

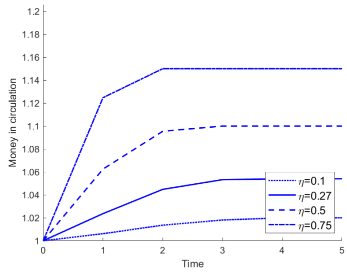
Notes: Connected (C) and unconnected (U) cases. For the Gini indexes, $G \in \{C, R, M\}$ stand for consumption, revenue, and money holdings.



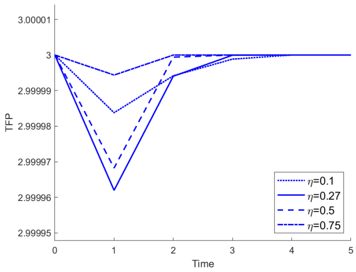
(a) Path of output, Y_t



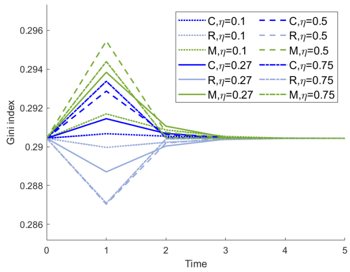
(b) Aggregate price, P_t



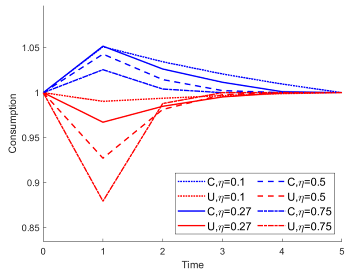
(c) Money in circulation, M_t^C



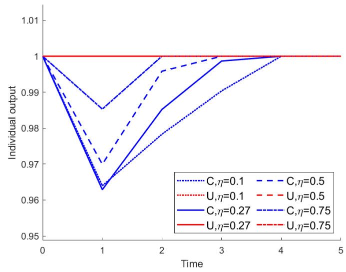
(d) Total Factor Productivity



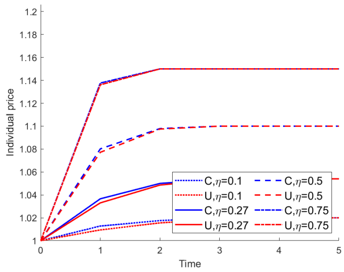
(e) Gini indexes



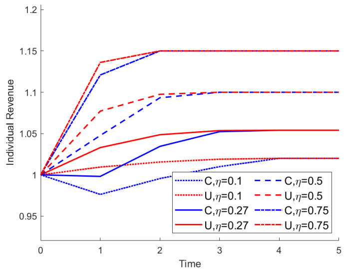
(f) Indiv. consumption, $C_{it}(\mathcal{Z})$



(g) Individual output, $y_{it}(\mathcal{Z})$



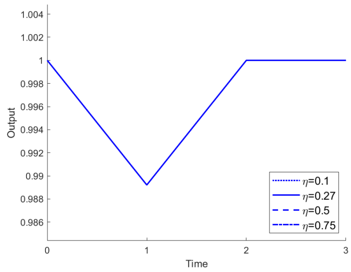
(h) Individual prices, $p_{it}(\mathcal{Z})$



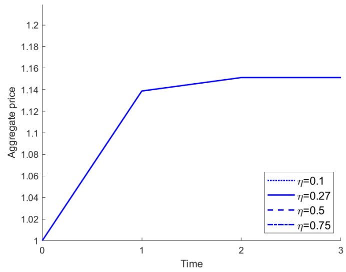
(i) Indiv. revenues, $R_{it}(\mathcal{Z})$

Figure 12: Paths for aggregate and individual variables for different values of η and τ^A

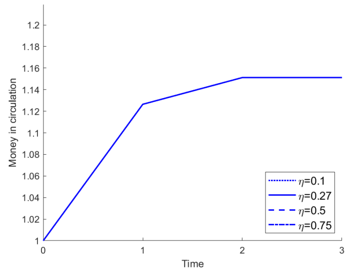
Notes: Connected (C) and unconnected (U) cases. For the Gini indexes, $G \in \{C, R, M\}$ stand for consumption, revenue, and money holdings.



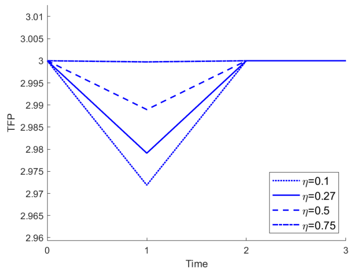
(a) Path of output, Y_t



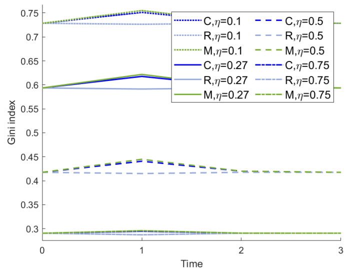
(b) Aggregate price, P_t



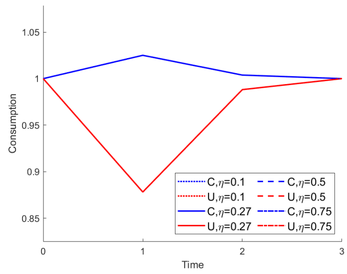
(c) Money in circulation, M_t^C



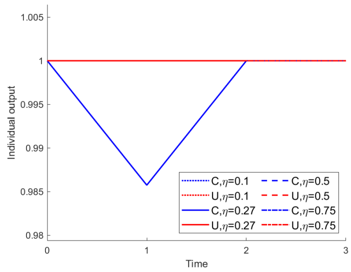
(d) Total Factor Productivity



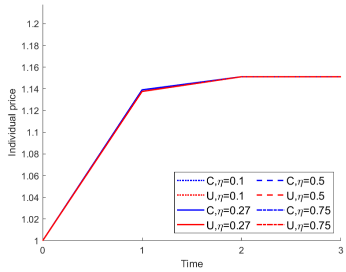
(e) Gini indexes



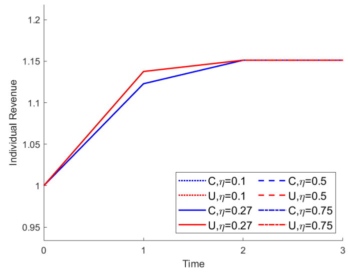
(f) Indiv. consumption, $C_{it}(\mathcal{Z})$



(g) Individual output, $y_{it}(\mathcal{Z})$



(h) Individual prices, $p_{it}(\mathcal{Z})$



(i) Indiv. revenues, $R_{it}(\mathcal{Z})$

Figure 13: Paths for aggregate and individual variables for different values of η and $\frac{M_{c0}}{M_0}$

Notes: Connected (C) and unconnected (U) cases. For the Gini indexes, $G \in \{C, R, M\}$ stand for consumption, revenue, and money holdings.

C Outstanding Tables

Quarter	ΔM_t
1999-Q4	10.0036%
2008-Q4	83.1813%
2009-Q4	12.4917%
2011-Q1	18.7556%
2011-Q2	10.5707%
2020-Q1	13.3256%
2020-Q2	28.8095%
2021-Q1	12.1483%
2022-Q2	-10.2375%

Table 3: Shocks to the U.S. monetary base of more than 10%

Source: Board of Governors of the Federal Reserve System (US), retrieved from FRED, Federal Reserve Bank of St. Louis

Fraction of Connected	Model	Constant output	No inequality
$M_{c0}/M_0 = 2.06$	-5.5304%	-5.3758%	-0.1853%
$M_{c0}/M_0 = 1$	-0.4016%	-0.217%	-0.1853%
$M_{c0}/M_0 = 0.6$	6.6595%	6.8873%	-0.1853%

The second column shows the welfare in the economy that does not receive a monetary shock. The third column contains the values for the welfare function under the benchmark specification of the baseline economy. The fourth column presents the counterfactual exercise of assuming that output is constant at the initial level, but keeping the degree of inequality across the connected and unconnected agents. The last column stands for the opposite exercise: it removes inequality between connected and unconnected agents with the same productivity but maintains the fall in output.

Table 4: Counterfactual welfare analysis of the baseline economy

Fraction of Connected	Model	Counterfactual
$M_{c0}/M_0 = 2.06$	-5.0228%	-0.1025%
$M_{c0}/M_0 = 1$	-0.0020807%	-4.14e-05%
$M_{c0}/M_0 = 0.6$	6.8995%	0.13286%

The second column shows the welfare in the economy that does not receive a monetary shock. The third column contains the values for the welfare function under the benchmark specification of the full enforcement economy. The counterfactual corresponds to the exercise of assuming that, after period T, as the baseline economy returns to equilibrium, the full enforcement economy returns as well.

Table 5: Counterfactual welfare analysis of the full enforcement economy

Fraction of Connected	Model	Baseline output	Baseline inequality	No inequality
$M_{c0}/M_0 = 2.06$	-5.0894%	-5.2367%	-6.0506%	0.0025%
$M_{c0}/M_0 = 1$	-0.1006%	-0.2876%	-0.2276%	0.0025%
$M_{c0}/M_0 = 0.6$	6.755%	6.5109%	7.8565%	0.0025%

The second column shows the welfare in the economy that does not receive a monetary shock. The third column contains the values for the welfare function under the benchmark specification of the zero interest rate economy. The fourth column stands for the exercise of keeping the degree of consumption and labor inequality in the zero interest rate, but imposing that the aggregate output be equal to the one in the baseline economy. The penultimate column stands for the opposite exercise: keeping the output level, but modifying consumption inequality between connected and unconnected. The last column stands for the counterfactual removal of inequality across connected and unconnected agents with the same productivity.

Table 6: Counterfactual welfare analysis of the zero interest rate economy

Model	$\eta = 0.1$	$\eta = 0.27$	$\eta = 0.5$	$\eta = 0.75$
Baseline	-0.9380%	-0.6945%	-0.4560%	-0.2130%
Full enforcement	-0.0234%	-0.0070%	-0.0026%	-0.0009%
Zero interest rate	-0.3315%	-0.1737%	-0.1101%	-0.0692%

Table 7: Welfare analysis for different values of η and τ

Model	$\eta = 0.1$	$\eta = 0.27$	$\eta = 0.5$	$\eta = 0.75$
Baseline	-0.1033%	-0.2591%	-0.3905%	-0.3418%
Full enforcement	-0.0009%	-0.0018%	-0.0022%	-0.0015%
Zero interest rate	-0.0276%	-0.0650%	-0.0971%	-0.1026%

Table 8: Welfare analysis for different values of η and τ^A

Model	T	$\frac{Y_1-Y_0}{Y_0}$	$\frac{P_1-P_0}{P_T-P_0}$	$\frac{C_{c1}(\mathcal{Z})-C_{u1}(\mathcal{Z})}{C_0(\mathcal{Z})}$	Consumption equivalent
Model	3	-1.521%	82.382%	12.359%	-5.530%
Negative Shock	3	-1.441%	119.983%	-10.810%	5.085%
			CRRA Utility		
$IES = 0.5$	2	-2.025%	79.267%	10.929%	-13.491%
$IES = 2$	2	-1.029%	89.662%	14.456%	-3.130%
			Inverse Frisch Elasticities		
$\zeta = 0$	2	-3.719%	99.946%	11.303%	-5.733%
$\zeta = 0.5$	3	-2.187%	88.549%	12.19%	-5.589%
$\zeta = 1$	3	-1.521%	82.382%	12.359%	-5.530%
$\zeta = 2$	3	-0.948%	77.318%	12.535%	-5.475%
			Access to Financial Markets and Fixed Aggregate Shock		
$\eta = 0.1$	8	-1.236%	29.691%	19.614%	-0.938%
$\eta = 0.27$	5	-1.850%	55.750%	14.776%	-0.695%
$\eta = 0.5$	3	-1.663%	77.867%	12.616%	-0.456%
$\eta = 0.75$	2	-0.768%	92.303%	11.391%	-0.213%
			Access to Financial Markets and Fixed Idiosyncratic Shock		
$\eta = 0.1$	4	-0.361%	48.849%	6.068%	-0.103%
$\eta = 0.27$	4	-1.003%	62.927%	8.455%	-0.259%
$\eta = 0.5$	3	-1.504%	78.666%	11.578%	-0.391%
$\eta = 0.75$	2	-1.103%	91.509%	14.622%	-0.342%

The columns show, from second to last: 2) the moment the economy returns to the stationary equilibrium; 3) the percentage variation in aggregate output; 4) the percentage variation in the aggregate price relative to the final stationary equilibrium; 5) distortion in consumption of high- to low-cash agents with average productivity relative to the stationary equilibrium; and 6) the short-run consumption equivalent.

In line with the text, the productivity is assumed to not differ between the average connected and unconnected agent. This is why the welfare losses are an order of magnitude lower than for the other cases.

Table 9: Summary of sensitivity exercises